Updated 05/23/05

Technical Mathematics 2

Technical Mathematics 2 continues students' study of algebra and geometry, building upon middle and high school topics. Geometry, functions, and statistical methods for estimation and prediction are the topics to be studied in an application-centered environment. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- ***Apply geometric properties and relationships to solve problems.***
- ***Use tables, formulas and algebraic expressions to model and solve problems.***
- ***Identify, graph, and use exponential and quadratic functions to solve problems.***
- ***Create linear models, for a set of data, to solve problems.***

NC DEPARTMENT OF PUBLIC INSTRUCTION

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Technical Mathematics 2

GOAL 1: The learner will use properties of geometric figures to solve problems.

1.01 Apply the properties and definitions of lines and angles to model and solve problems.
1.02 Apply the properties and definitions of plane figures to model and solve problems:
- a) Triangles.
- b) Quadrilaterals.
- c) Other polygons.
- d) Circles.

GOAL 2: The learner will use relations and functions to solve problems.

2.01 Use quadratic equations to model and solve problems; justify results.
- a) Solve using tables, graphs, and algebraic properties.
- b) Interpret the constants and coefficients in the context of the problem.

2.02 Use exponential functions to model and solve problems; justify results.
- a) Solve using tables, graphs, and algebraic properties.
- b) Interpret the constants, coefficients, and bases in the context of the problem.

2.03 Create, interpret, and analyze best-fit models of linear, exponential, and quadratic functions to solve problems.
- a) Interpret the constants, coefficients, and bases in the context of the data.
- b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

2.04 Use systems of equations and inequalities to model and solve problems using tables, graphs, matrix operations, and algebraic properties; justify results.

Vocabulary Concepts Skills

Adjacent Angles

Alternate Interior

Bisector

Collinear

Complementary

Congruence

Constructions

Corresponding

Length

Linear Pair

Midpoint

Parallel

Perpendicular

Skew

Slope

Supplementary

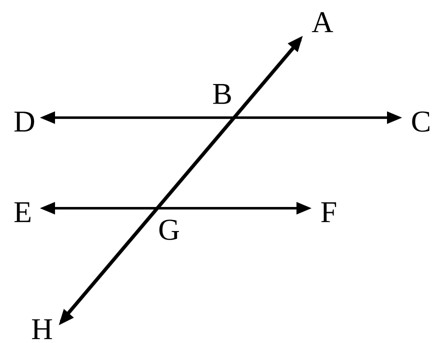
Transversal

Vertical

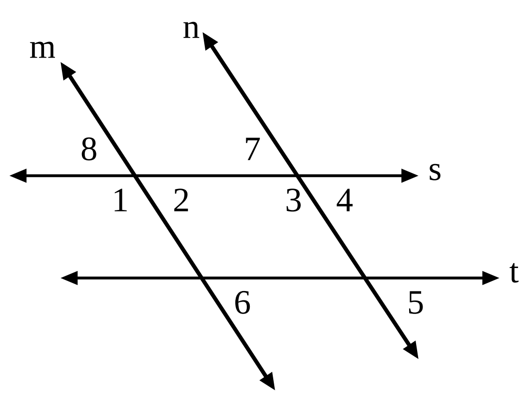
1.01 Apply the properties and definitions of lines and angles to model and solve problems.

A. If SU bisects ∠RST, m∠RSU = (2*x* - 11), and m∠RST = (3*x* + 23), find m∠TSU. Explain your result.

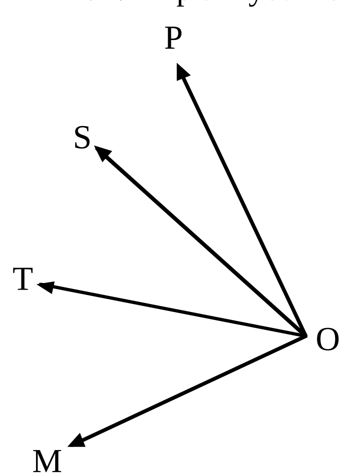
C. $\overleftrightarrow{DC}$ and $\overleftrightarrow{EF}$ are parallel, m∠EGH = (2*x* - 5), and m∠GBC = (3*x* - 10). Determine the m∠ABC. Explain your reasoning.



D. Given ∠1 ≅ ∠3 and ∠7 ≅ ∠6, are $\overleftrightarrow{s}$ and $\overleftrightarrow{t}$ parallel? Explain your answer.

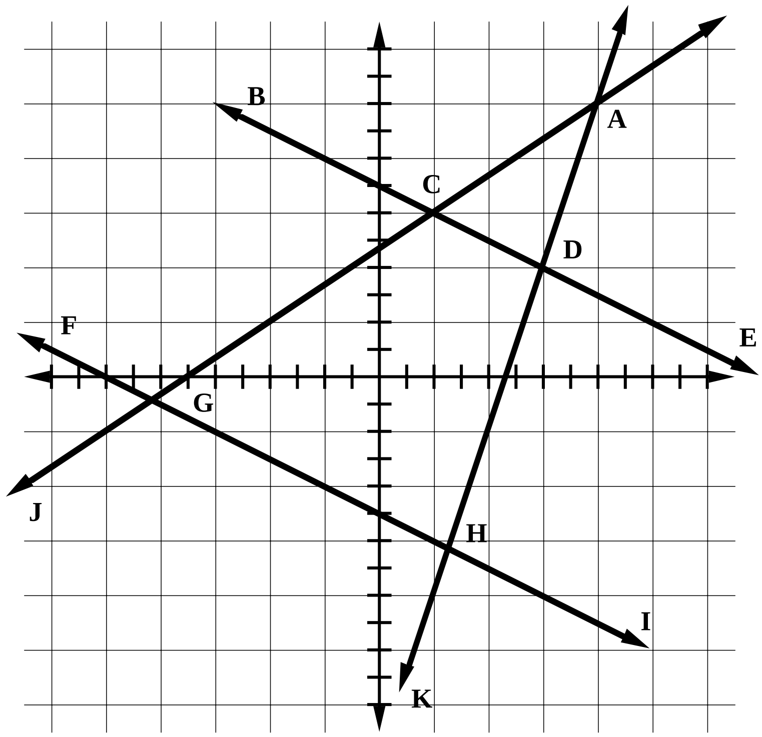


G. Given $\overrightarrow{OS}$ bisects ∠TOP, $\overrightarrow{OM} \perp \overrightarrow{OP}$, m∠MOT = (3*x* + 3), and m∠TOS = (2*x* + 5), find m∠TOP. Explain your results.

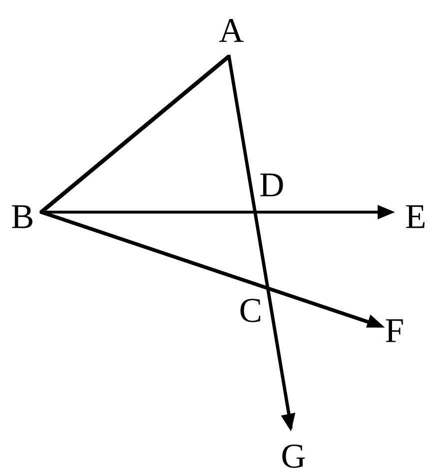


Vocabulary Concepts Skills

E. Find m∠FGJ, m∠KHI, and m∠CAD to the nearest hundredth. Explain your results.



F. Find m∠DBC if m∠ACB = (21*x* + 36), m∠FCG = (13*x* + 42), and m∠ADB = 83. Explain your results.



Vocabulary Concepts Skills

Equilateral

Isosceles

Altitude

Perpendicular Bisector

Median

Congruence

Similarity

Corresponding Parts

Triangle Inequality

Constructions

Law of Sines

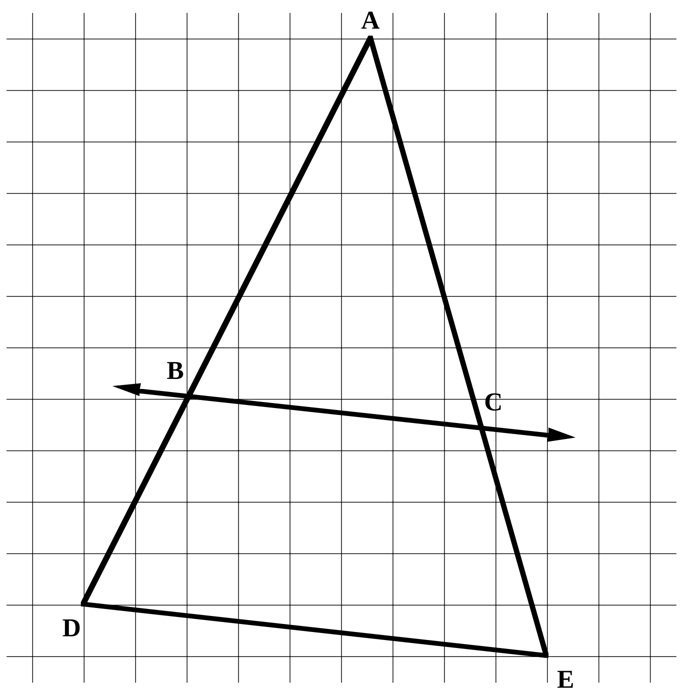
Law of Cosines

Pythagorean Theorem

1.02 Apply the properties and definitions of plane figures to model and solve problems.

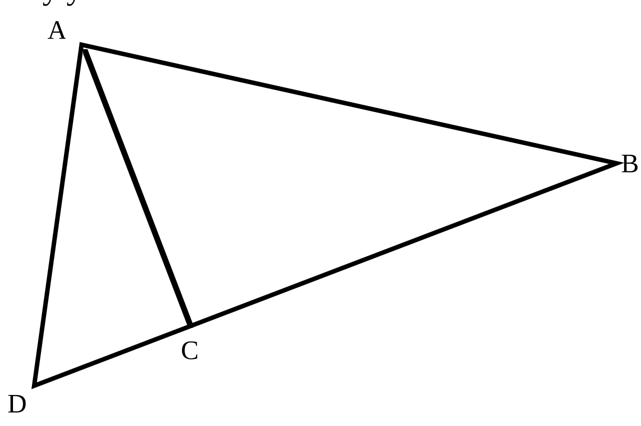
a) Triangles

A. $\overleftrightarrow{BC}$ and $\overline{DE}$ are parallel. Find the perimeters of ABC and BCED. Justify your results.



B. In ΔABC, AB = 27 and BC = 15. What is true about the length of $\overline{AC}$? Explain.

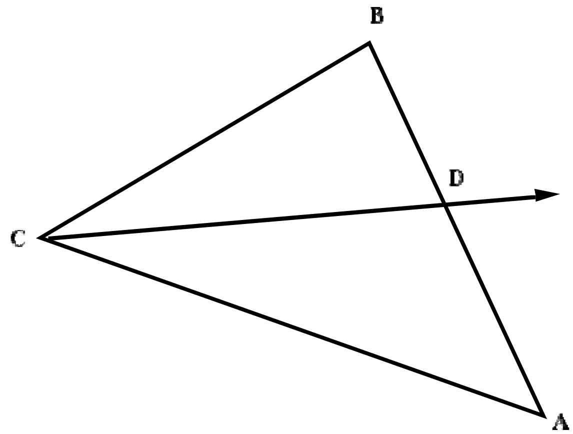
C. In ΔADB, m∠DAB = m∠ACB = 90, AB = 19, and AC = 9. Find DC. Justify your results.



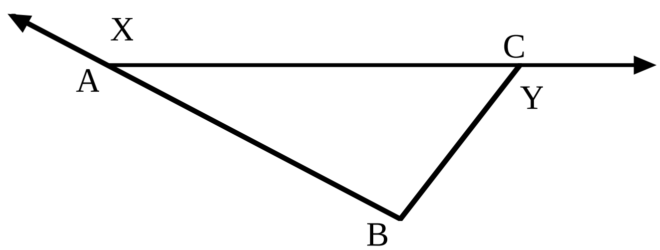
Vocabulary Concepts Skills

D. ΔABC is equilateral with vertices A (5, 3) and B (10, 8). Locate vertex C. Explain.

E. $\overrightarrow{CD}$ is the bisector of ∠ACB, m∠A = 46 and m∠B = 82. Find m∠ACD. Justify your results.



F. In ΔABC, $\overline{AB} \cong \overline{BC}$. If m∠Y = 112, what is the measure of ∠X? Justify your results.



G. In ΔSAT, m∠S = (2*x* - 10), m∠A = (*x* + 15), and m∠T = (4*x* - 20). Describe ΔSAT.

H. The sides of ΔPQT are 17.6, 11.7, and 9.6 meters. Find the perimeter of the triangle formed by connecting the midpoints of the sides of ΔPQT. Justify your results.

Vocabulary Concepts Skills

Base

Congruence

Consecutive Angles

Constructions

Diagonal

Height

Isosceles Trapezoid

Kite

Midsegment (median)

Opposite Angles

Opposite Sides

Parallel

Parallelogram

Perpendicular

Rectangle

Rhombus

Similarity

Slope

Square

Trapezoid

1.02 Apply the properties and definitions of plane figures to model and solve problems.

b) Quadrilaterals

A. The vertices of ABCD are A(-5, 1), B(3, 6), C(7, 0), and D(-1, -5). What is the best name for the figure? Justify your answer.

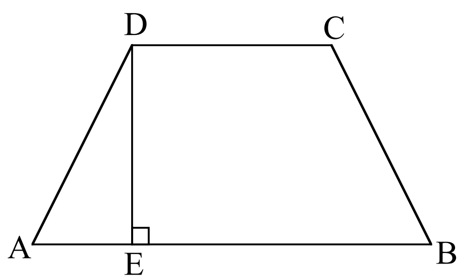
B. Find the length of the midsegment of the trapezoid with vertices (-3, -2), (-2, 1), (4, 5), and (1, -4). Is the trapezoid isosceles? Justify.

C. For parallelogram ABCD, m∠A = (8*x* - 16) and the measure of the exterior angle at C is (5*x* + 18). Find m∠B; justify.

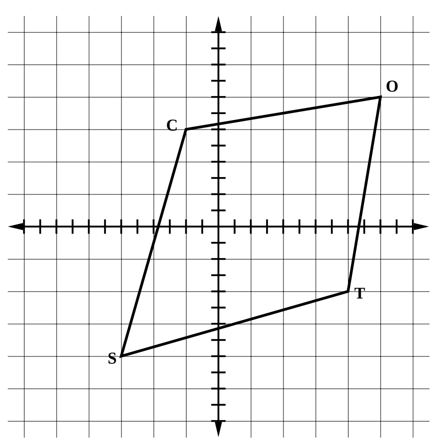
D. In parallelogram BCDF with A(3, -1) and D(-1, 5), find the coordinates of the point of intersection of the diagonals. Explain.

E. A parallelogram has vertices (-4, 5), (-1, -4), and (6, 4). What are the ordered pairs that can be the fourth vertex? Justify each.

F. In the diagram of isosceles trapezoid ABCD, m∠A = 53, DE = 6, and DC = 10. Find the perimeter of ABCD to the nearest tenth. Justify your results.



G. Which quadrilateral is TOCS? Justify.



Vocabulary
Concepts
Skills

Apothem

Circumscribed

Composite

Concave

Congruence

Convex

Diagonal

Equiangular

Equilateral

Exterior Angles

Inscribed

Interior Angles

Irregular

Prefixes (*n*-gons)

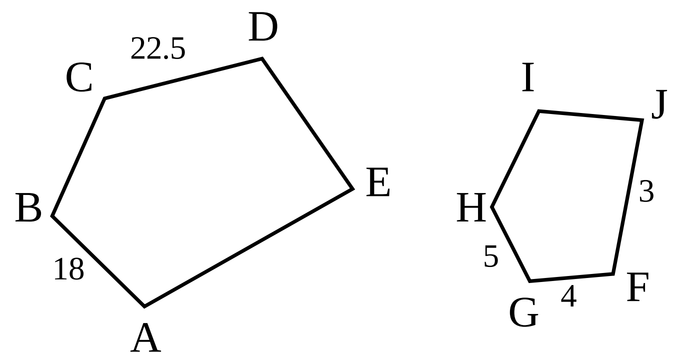
Regular

Similarity

1.02 Apply the properties and definitions of plane figures to model and solve problems.
c) Other Polygons

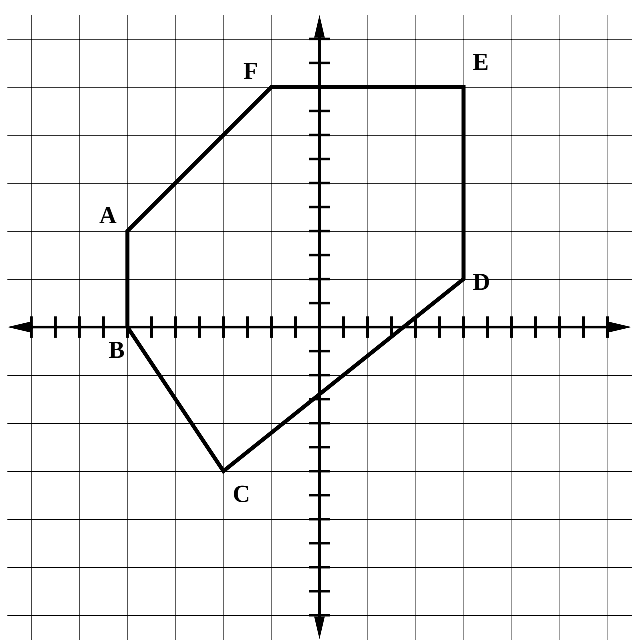
A. The measure of an exterior angle of a regular polygon is 20^o. Find the number of sides of the polygon. Explain.

B. ABCDE ~ FGHIJ. Find the perimeter of each figure. Explain.



C. In convex pentagon ABCDE m∠A = 6*x*, m∠B = (4*x* + 13), m∠C = (*x* + 9), m∠D = (2*x* - 8), and m∠E = (4*x* - 1). What are the measures of all the angles? Justify your results.

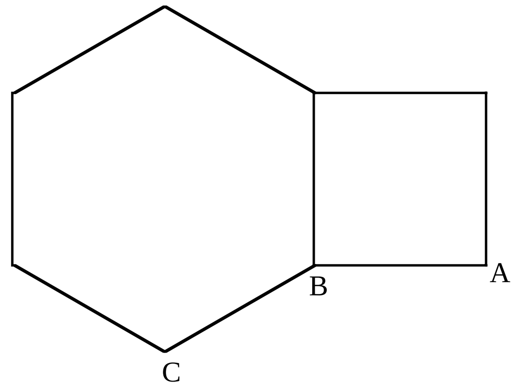
D. Find the measures of all the exterior angles of ABCDEF. Justify your results.



Vocabulary
Concepts
Skills

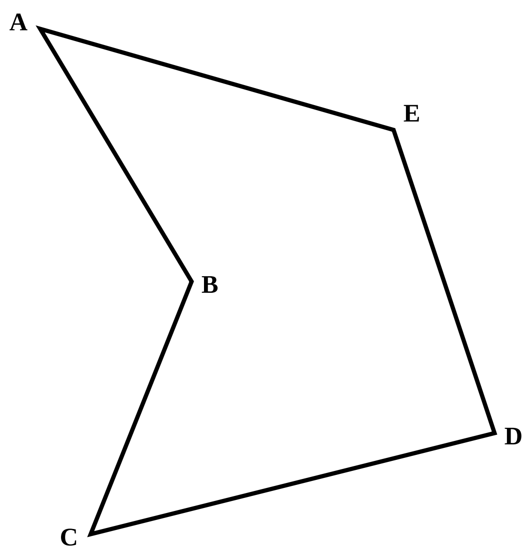
E. Find the apothem of a regular hexagon with sides of length 7.6 cm. Explain.

F. Point B is a mutual vertex of a regular hexagon, a square, and a third regular polygon (not completely shown). If two of the sides of this third polygon are $\overline{AB}$ and $\overline{BC}$, what is this polygon? Justify.

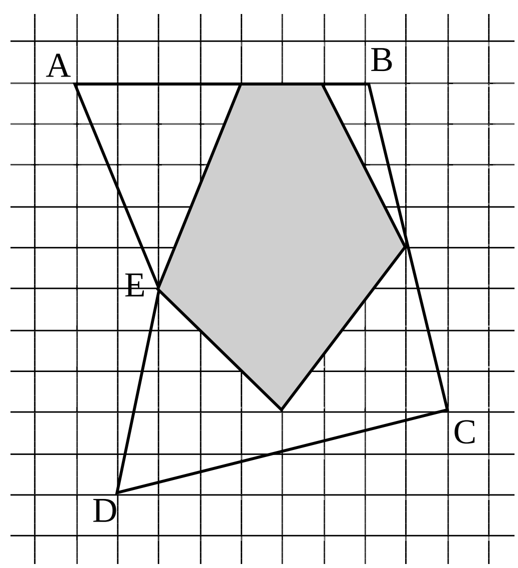


G. ABCDEF is a convex figure where $\angle A \cong \angle D$, $\angle B \cong \angle C \cong \angle E \cong \angle F$, and the measure of the exterior angle at E is 8°. What are the measures of all the interior angles?

H. Relocate vertex B so that ABCDE is convex and all sides remain the same length. Explain.



I. What is the probability that a point in the interior of ABCDE is in the unshaded area?



Vocabulary Concepts Skills

Arc

Center

Central Angle

Chord

Circumference

Circumscribed

Concentric

Constructions

Diameter

Inscribed

π

Radius

Secant

Sectors

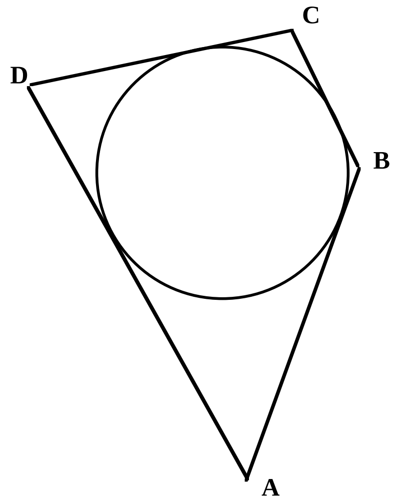
Segments

Tangent

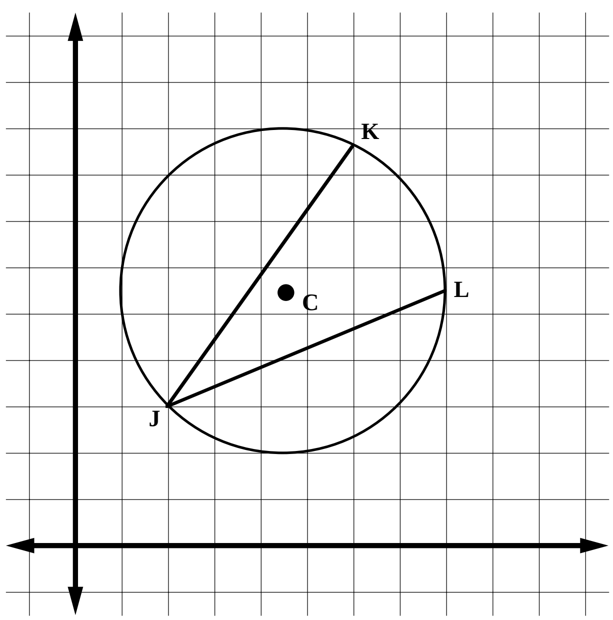
1.02 Apply the properties and definitions of plane figures to model and solve problems.

d) Circles

A. $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$ are tangent to the circle, AB = 14, BC = 12, and CD = 16. Find DA. Explain.

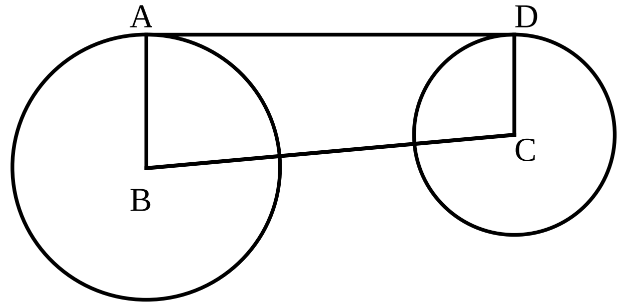


B. Given circle C, find the length of $\overset{\frown}{KL}$ and m∠KJL. Justify your results.

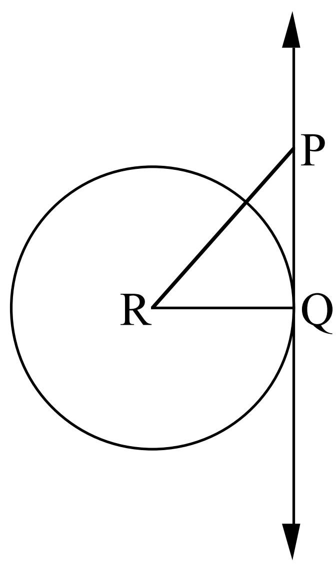


Vocabulary Concepts Skills

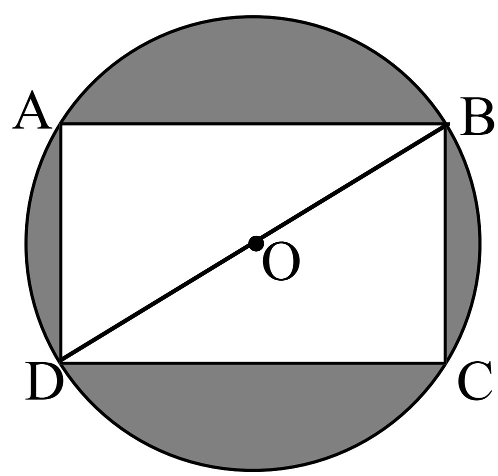
C. The radius of circle B is 38, the radius of circle C is 24 and AD = 91. Find BC. Explain.



D. $\overleftrightarrow{PQ}$ is a tangent to circle R at point Q. The circle has a radius of 19. If m∠R = 50, find RP. Justify your results.



E. In the diagram, ABCD is a rectangle inscribed in the circle O. The ratio AB to BC is 7:3. The area of the rectangle is 165 cm^2. Find the area of the shaded portion.



Vocabulary Concepts Skills

Domain

Range

Graph

Parabola

Maximum

Minimum

Zeros

Intercepts

Axis of Symmetry

Quadratic Formula

$y = ax^2 + bx + c$

$y - k = a(x - h)^2$

2.01 Use quadratic equations to model and solve problems; justify results.

A. For $f(x) = 2x^2 - 17x + 26$, locate the vertex and intercepts. Justify the results algebraically.

B. Solve $3x^2 - 5 = 7x$. Justify the results algebraically.

C. Solve $x^2 + 3x = 28$. Justify the results algebraically.

D. For $f(x) = ax^2 - 2x + 5$: As **a** gets close to zero, how does the graph of $f(x)$ change? For $f(x) = 2x^2 + bx + 10$: As **b** increases/decreases, how does the graph of $f(x)$ change? For $f(x) = 2.5x^2 + 17x + c$: As **c** increases/decreases, how does the graph of $f(x)$ change?

E. The function $f(x) = -0.019x^2 + 3.04x - 58.87$ describes newspaper circulation (millions) in the United States for 1920-98 ($x = 20$ for 1920). Identify periods of increasing and decreasing circulation. According to the function, when did newspaper circulation peak? When will circulation approximate 45 million?

F. The space shuttle uses solid rocket boosters (SRB) during the launch phase of its flight from Cape Canaveral. The SRBs burn for about two minutes, shut down, detach from the main rocket assembly, and fall back to Earth 140 miles downrange. Parachutes assist the ocean landing, beginning at an altitude of 20,000 feet. The SRBs are recovered and used again for a later launch. The function $f(x) = -33 + 0.72x – 0.00176x^2$, for $x \geq 115.5$, describes the altitude of the shuttle's SRBs since the launch (x is elapsed time in seconds). How long after launch do the SRBs splashdown? The SRBs separate from the shuttle after 115.5 seconds. How much longer do the SRBs continue to gain altitude?

G. The height of a baseball, just hit, is approximated by $h(x) = -0.0011x^2 + 0.4545x + 3$ where x is the horizontal distance from homeplate. The ball is approaching the ten-foot outfield wall, 400 feet from homeplate. If the ball is not caught, will it go over the wall?

H. Each orange tree in a California grove produces 600 oranges per year if no more than 20 trees are palnted per acre. For each additional tree planted per acre, the yield per tree decreases by 15 oranges. How many trees per acre should be planted to obtain the greatest number of oranges?

Vocabulary
Concepts
Skills

Domain

Range

Rate

Factor

Base

Initial Value

$f(x) = ab^x$

$f(x) = a(1 + r)^x$

Decay

Growth

Compound Interest

Half-Life

2.02 Use exponential functions to model and solve problems; justify results.

A. Bacteria growing on disgarded food triples every five hours. If there are one million bacteria present now, how many will there be one day later?

B. At the end of four years (t), a savings account paying 5.35% annually (r) compounded continuously, had a balance (B) of $3096.56. What was the initial deposit (P)? (Use $B = Pe^{rt}$) If the initial deposit had been in an account compounded annually, how much less interest would have been earned?

C. Over the last year the stock value of an internet company has dropped at a rate of 17% per month. The value of the stock at the beginning of the year was $19.50. What was the value of the stock at the end of the year? If the stock's value continues to decrease at the same rate, how long does it take the stock to be worth one-tenth of its original value?

D. In 1998 there were 429,316 people employed in the United States as computer support specialists. According to the Bureau of Labor Statistics, that number is expected to grow to 868,674 by 2008. Assuming a constant annual growth rate in the number of specialists, how many will there be in 2006? Assuming the growth continues, when will the number of computer support specialists exceed one million? Give the algebraic model for this growth.

E. For $f(x) = 5 \cdot b^x$ and $b > 1$: As **b** increases, how does the graph of $f(x)$ change?

F. For $f(x) = 5 \cdot b^x$ and $0 < b < 1$: As **b** approaches zero, how does the graph of $f(x)$ change?

G. For $f(x) = a \cdot 1.9^x$ and $a \geq 1$: As **a** increases, how does the graph of $f(x)$ change?

Vocabulary
Concepts
Skills

Independent (Explanatory)

Dependent (Response)

Domain

Range

Scatter Plot

Correlation Coefficient

Residuals

Estimation

Prediction

Interpolation

Extrapolation

Intercepts

Slope

2.03 Create, interpret, and analyze best-fit models of linear, exponential, and quadratic functions to solve problems.

A. The winnings for the 2003 *Coca-Cola 600* are shown. Create an algebraic model of the data. Describe the average change in winnings and how it relates to the data. What are other variables which affect the prizes at an automobile race or any other professional athletic event?

Place	**Winnings**	**Place**	**Winnings**
1	271,900	9	129,153
2	206,500	10	120,331
3	184,633	11	84,740
4	137,350	12	117,542
5	186,850	13	119,003
6	112,875	14	104,600
7	132,225	15	110,950
8	135,103	16	76,500

B. Graph and describe the newspaper circulation (in millions) data shown. What variables affect newspaper circulation? Create an algebraic model of the data (let x = 20 for 1920). According to the model, will the newspaper circulation drop below 50 million? If it does, when?

1920	27.8	1960	58.9	1992	60.2
1925	33.7	1965	60.4	1993	59.8
1930	39.6	1970	62.1	1994	59.3
1935	38.2	1975	60.7	1995	58.2
1940	41.1	1980	62.2	1996	57.0
1945	48.4	1985	62.8	1997	56.7
1950	53.8	1990	62.3	1998	56.2
1955	56.1	1991	60.7		

C. Determine the algebraic expressions that model the surface areas and volumes of the polyhedral patterns on B-40 and B-41 in ***Resources for Algebra***.

Vocabulary
Concepts
Skills

D. Since the 1924 Olympics, men and women have competed separately in 400 Meter Free Style swimming events. Compare the data (all results in seconds) for the two events and describe similarities and differences. Why would the number of total participants be considered a variable in estimating athletic performance? Identify and discuss other variables that affect athletic performance. Determine best-fit models for each of the men's and women's data. Which group of athletes is making the greater improvement? Explain. Assuming winning performances occur according to the models, what will be the results for the 2004 Athens Olympics? According to the models, will the women's performance ever equal or exceed the men's performance in this event? If so, when? If not, why? Include other data to support your hypothesis.
Does the number of total participants correlate well with athletic performance? Explain. Identify and discuss variables that affect athletic performance.

Performance	**Men**	**Women**
1924	304.2	362.2
1928	301.6	342.8
1932	288.4	328.5
1936	284.5	326.4
1948	281.0	317.8
1952	270.7	312.1
1956	267.3	294.6
1960	258.3	290.6
1964	252.2	283.3
1968	249.0	271.8
1972	240.27	259.04
1976	231.93	249.89
1980	231.31	248.76
1984	231.23	247.10
1988	226.95	243.85
1992	225.00	247.18
1996	227.95	247.25
2000	220.59	245.80

TotalParticipation	**Men**	**Women**
1924	2956	136
1928	2724	290
1932	1281	127
1936	3738	328
1948	3714	385
1952	4407	518
1956	3003	397
1960	4738	610
1964	4457	683
1968	4750	781
1972	6659	1171
1976	4915	1274
1980	4320	1192
1984	5458	1620
1988	6983	2438
1992	7555	3008
1996	7060	3684
2000	6582	4069

Vocabulary Concepts Skills

Domain

Range

Graph

Intersection

Linear Programming

Maximum

Minimum

Constraints

Parameters

Margin of Error

Sustitution

AX = B

2.04 Use systems of equations and inequalities to model and solve problems using tables, graphs, matrix operations, and algebraic properties; justify results.

A**.** For the period 1970-1998 ($x = 0$ for 1970), $f(x) = 0.71x^2 + 2.15x + 67.53$ models US exports and $g(x) = 0.82x^2 + 6.42x + 55.07$ models US imports. Find the years when US trade was balanced, $f(x) = g(x)$. Graphically identify and algebraically define the US trade surplus/deficit, according to the functions provided.

B. A luxury car's value is represented by the equation $y_1 = 50(0.822)^x$. A sports utility vehicle's (SUV) value is represented by the equation $y_2 = 30(0.884)^x$. In both functions, $\boldsymbol{y}$ is the value ($1000's) of the automobile after $\boldsymbol{x}$ years. Assuming one of each model is purchased on the same day, how long before the luxury car is only worth $1000 more than the SUV? How long until the two automobiles are equal in value? How long before the SUV is worth $1000 more than the luxury car?

C. Write the system of inequalities that describes a triangular region with one vertex at (7, 5) and another on the x-axis. None of the sides can be horizontal or vertical. (3.13)

D. Find the solution for the system:

$$6x - 3y + 19z = 23$$
$$-5x + 11y - 5z = -14$$
$$x + 7y + 4z = 6$$

E. Find the solution for the system:

$$-w + 2.4x + 4.5y - 7.1z = 19$$
$$3w + 7.7x - 5.8y + 2z = 0$$
$$1.3w - 4.3x + 9.2y - z = 23.4$$
$$-5.3w + 6.6x + 1.6y + 7.9z = 4.4$$

F. Write a system of inequalities that defines a parallelogram in the second quadrant with one vertex at (-3, 4). The sides cannot be vertical or horizontal.

Vocabulary
Concepts
Skills

G. During the summer break, Nathan works as many as 35 hours per week. On Saturdays he spends between two and six hours delivering furniture. On weekdays he can work between 10 and 40 hours at the recreation center as a counselor for the children's day camp. Delivering furniture pays $10 per hour while the summer counselor's job pays $6.25 per hour. State the system of inequalities which describe Nathan's summer work, give the algebraic expression for his weekly income, graph the system of inequalities, and find the maximum weekly income.