Updated 05/23/05

Technical Mathematics 1

Technical Mathematics 1 continues students' study of algebra and geometry, building upon middle school and Algebra 1 topics. Problem solving, measurement, special relationships in right triangles, transformations, and geometric applications of algebra are the topics to be studied in an application-centered environment. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- ***Apply geometric properties and relationships to solve problems.***
- ***Use formulas and algebraic expressions to model and solve problems.***
- ***Define and use linear functions to model and solve problems.***
- ***Operate with matrices to model and solve problems.***

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Technical Mathematics 1

GOAL 1: The learner will apply various strategies to solve problems.

1.01 Apply various techniques and strategies to solve problems.
a) Select or create an appropriate graphical display for a given set of data.
b) Identify and represent patterns using appropriate algebraic notation.
c) Select and apply appropriate formulas.
d) Choose or create appropriate representations of two- and three-dimensional figures.

GOAL 2: The leaarner will measure and apply geometric concepts to solve problems.

2.01 Select and use appropriate tools to measure two- and three-dimensional figures; interpret and communicate results with regard to precision.
2.02 Interpret and construct maps and scale drawings to solve problems.
2.03 Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.
2.04 Use the trigonometric ratios to model and solve problems involving right triangles.

GOAL 3: The learner will describe the transformation of polygons in the coordinate plane geometrically.

3.01 Apply algebraic and trigonometric concepts to confirm properties of geometric figures in the coordinate plane.
3.02 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.
3.03 Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

Vocabulary
Concepts
Skills

Diagram

Systematic Lists

Elimination

Matrix Logic

Patterns

Formulas

Algebraic Notation

Guess and Check

Unit Analysis

Work Backwards

Venn Diagrams

Two- and Three Dimensional Representations

Finite Differences

1.01 Apply various techniques and strategies to solve problems.

a) Select or create an appropriate graphical display for a given set of data.

b) Identify and represent patterns using appropriate algebraic notation.

c) Select and apply appropriate formulas.

d) Choose or create appropriate representations of two- and three-dimensional figures.

A. Johnny is going to build a swimming pool. The dimensions of the pool are 14 feet by 26 feet. He wants a rectangular concrete deck around the pool that is six feet on all sides. Draw a diagram and explain how to find the area of the deck.

B. List the ways your school's football team can score 21 points in a game. Points are scored as follows: A safety scores 2 points, a field goal 3 points, a touchdown 6 points, and a point after touchdown (PAT) 1 point.

C. Without a calculator, find the fourth root of 923,521, an integer.

D. Fred, Kent, Allie, and Jane (two sets of siblings) each have a favorite sport: running, swimming, biking, and golf. Given the following clues, construct a matrix to help determine who likes which sport.
Fred hates golf. He agrees with Mark Twain that golf is nothing but a good walk spoiled. Kent wouldn't run around the block if he didn't have to, and neither would his sister. Each sister's favorite sport is featured in a triathlon. Allie bought her brother a new bike for his birthday to use in his favorite sport.

Vocabulary
Concepts
Skills

E. A pair of adult rabbits is purchased on April 1, 2002 and they produce another pair monthly after they are 2 months old. If each pair of offspring reproduce in the same manner, how many pairs of rabbits will there be on April 1, 2003? Make a table or list showing the monthly population.

F. Samantha and Justin have lots of stickers. Samantha had 1/3 as many as Justin had, but then Justin gave her six stickers, so now Samantha has half as many as Justin. How many stickers did each one start with?

G. A vehicle can travel 360 miles on a tank of gas. If the tank is completely empty, it costs $20.25 to fill up the tank. The gas tank capacity of the vehicle is 15 gallons. Determine each ratio and its reciprocal: miles per gallon, dollars per gallon, and dollars per mile. Identify a situation when each might be useful information.

H. A number between one and ten was selected. It is multiplied by seven. The product is added to three. The sum is divided by six. Eight is subtracted from the quotient. Three is the result. Explain how to find the original number.

I. In a poll of 46 students, 23 liked country music, 24 liked bluegrass, and 19 liked rock music. Of all the students 12 liked country and rock, 13 liked country and bluegrass, and 14 liked rock and bluegrass. Of those students, 9 liked all three types of music. How many students did not like any of these types? Explain your answer using a Venn diagram.

J. Use finite differences to determine an equation for the number of squares on any size checkerboard (n-by-n).

K. For any 4-sided convex polygon, 2 distinct diagonals can be drawn. For any 5-sided convex polygon, 5 distinct diagonals can be drawn. For any 6-sided convex polygon, 9 distinct diagonals can be drawn. How many distinct diagonals can be drawn in a 20-sided polygon? Use finite differences to determine the function that will generate the number of distinct diagonals for n-sided polygons.

2.01 Select and use appropriate tools to measure; interpret and communicate results with regard to precision.

Vocabulary
Concepts
Skills

Customery Units of Measure

Metric Units of Measure

Accuracy

Tolerance

Precision

Significant Digits

Margin of Error

Scientific Notation

Conversions between Systems of Measures

Vocabulary Concepts Skills

Ratios

Proportions

Direct Variation

vectors

Coordinate Systems

Similarity

Scale Factor

Percent

Perspective (side, top, front, corner)

Legend

Construction

2.02 Interpret and construct maps and scale drawings to solve problems.

A. Use a North Carolina map to answer the following questions. Locate and describe the three **best** routes from Hickory to Sanford. Limit your travel to interstate and US routes. Which is the most direct? Which maximizes interstate travel? Assume an average of 64 mph on all interstates and 53 mph on US routes. How long would each of the three routes take? There is a wreck between Winston Salem and Greensboro which causes traffic to stop completely for 30 minutes and further slows traffic to 24 mph for 12 miles of interstate travel. How does this situation affect travel time on your three routes? Is there now a fourth route that may be better? If so, describe the new route.

B. Have students draw a scale map of their community and include their principle route from home to school. Have them include landmarks, US/ state/county roads, topography, etc.

C. Have students make scale drawings of the classroom (a floor plan as well as views for each wall), including desks, file cabinets, bookshelves, etc.

Vocabulary Concepts Skills

Side

Face

Edge

Vertex

Perimeter

π

Circumference

Suface Area

Lateral Area

Volume

Polyhedron

Composite Figures

Truncated Figures

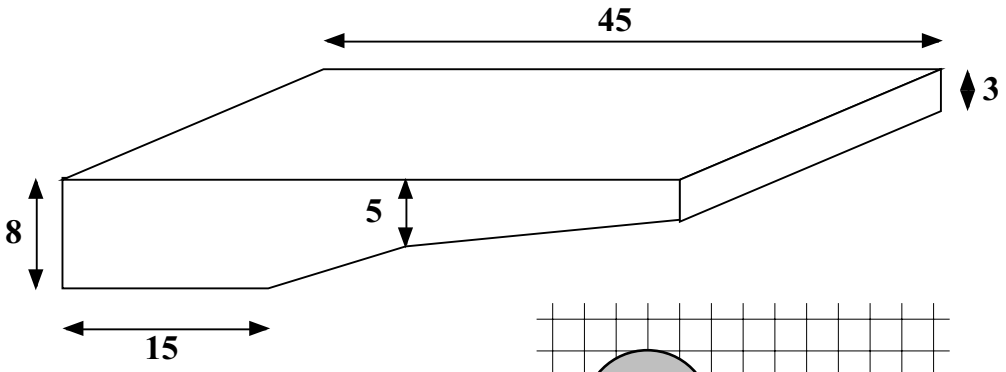
Net

Appropriate Units of Measure

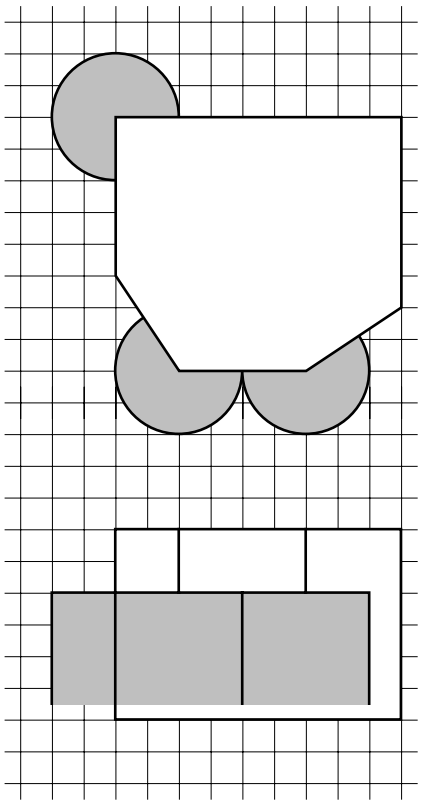
DRAFT

2.03 Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.

A. The pool shown (dimensions in feet) is filled with water. Water is leaking from the pool at the rate of 0.45 cubic feet per minute. At this rate, how many hours will it take for the water level to drop six feet?



B. The top and side views of the new museum are shown. On the grid, 1unit = 3 m. How much space will be heated or cooled during the year?



Vocabulary
Concepts
Skills

Sector

Arc

Prism

Pyramid

Cylinder

Cone

Sphere

Cube

Faces

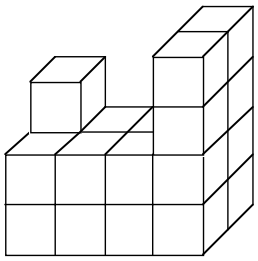
Vertices

Base

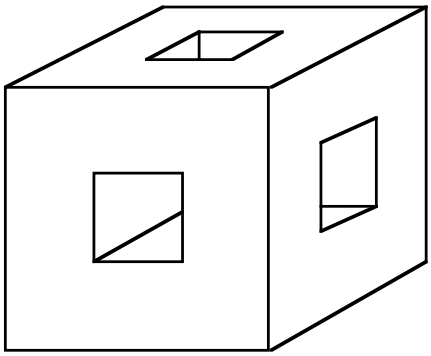
Edge

C. A cylindrical pipe has an outside radius of 6.5 inches and an inside radius of 5.3 inches. The pipe is six feet long. To the nearest tenth, how much total surface is exposed?

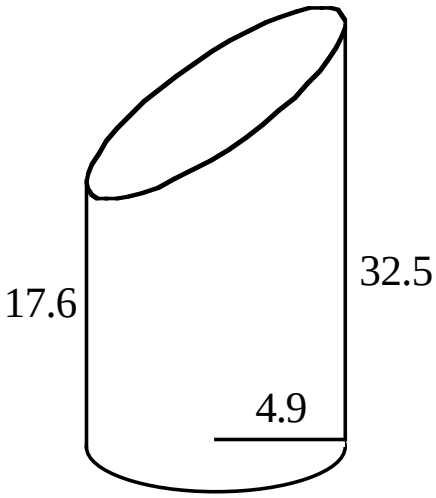
D. The figure shown was built with cubes. The bottom horizontal edge of the figure is 14 cm long. What is the volume of the figure? What is its surface area?



E. The plastic cube shown originally had a volume of 2500 cm^3. The front face is drawn to proportion. Square holes were cut through to the opposite face. How much surface is exposed?



F. A glass cylinder is cut as shown. Find the volume and total surface area. The area of the ellipse is πRr where R is the length of the semi-major axis or, in this case, half the length of the diagonal cut.



Vocabulary
Concepts
Skills

Right Triangle

Hypotenuse

Legs

Altitude

Sin A

Cos A

Tan A

Sin-1 A (Arcsine)

Cos-1 A (Arccosine)

Tan-1 A (Arctangent)

45^o-45^o-90^o Triangle

30^o-60^o-90^o Triangle

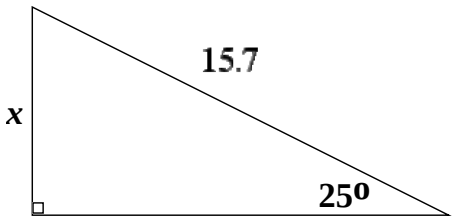
Angle of Elevation

Angle of Depression

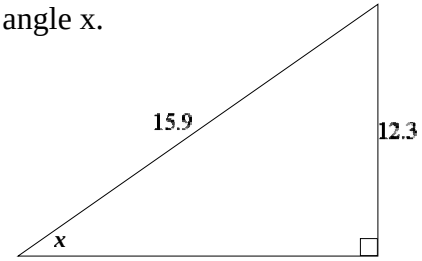
DRAFT

2.04 Use the trigonometric ratios to model and solve problems involving right triangles.

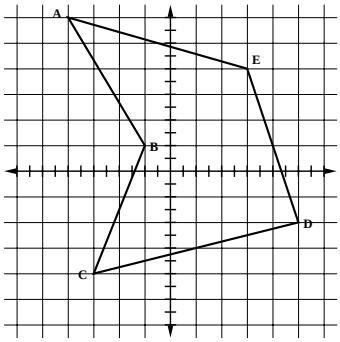
A. Find x.



B. Find the measure of angle x.



C. Find the measures of interior ∠CDE and exterior ∠ABC.



D. From the top of a building 50 feet high the angles of elevation and depression of the top and bottom of another building are 19.7^o and 26.6^o, respectively. What is the height of the second building and how far away is it?

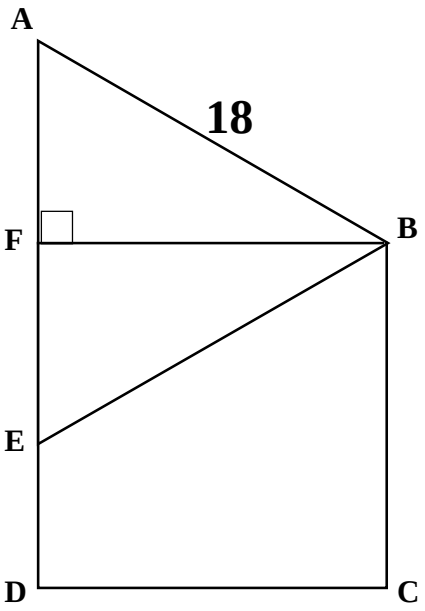
E. At two tracking stations ten miles apart, the elevation angles of a passing airliner are 16.5^o and 38.3^o, respectively. At what altitude is the airliner flying?

F. As a balloon passes between two points and 2 miles apart, the angles of elevation of the balloon at these points are 27.3^o and 41.8^o, respectively. Find the altitude of the balloon.

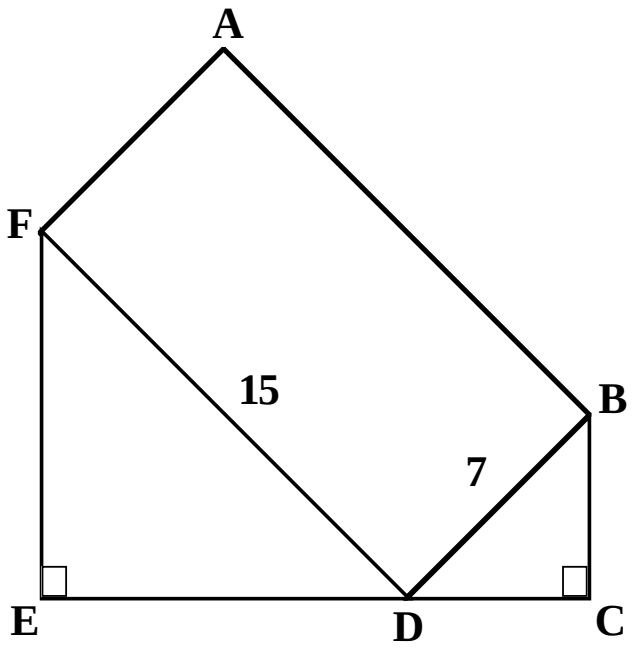
G. The top of a lighthouse is 230 feet above the sea. How far away is an object which is just "on the horizon"? (Assume the earth is a sphere of radius 3956 miles.) What must be the elevation of an observer in order that she may be able to see an object on the earth thirty miles away?

Vocabulary
Concepts
Skills

H. ABE is an equilateral triangle and BCDF is a square. What is the perimeter of BCDE? What is the area of BCDE?



I. ABDF is a rectangle and both ΔDEF and ΔBCD are isosceles. What is the perimeter of ABCEF? What is the area of ABCEF?



Vocabulary
Concepts
Skills

Linear Equations

Parallel

Perpendicular

Slope

Length

Distance Formula

Pythagorean Theorem

Vertex

Mid-Point

Area

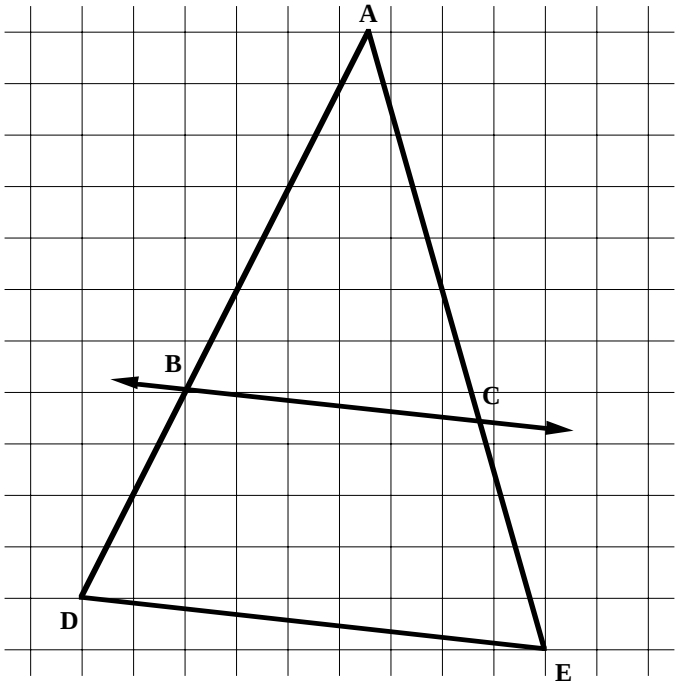
Angle Measure

Trigonometric Ratios

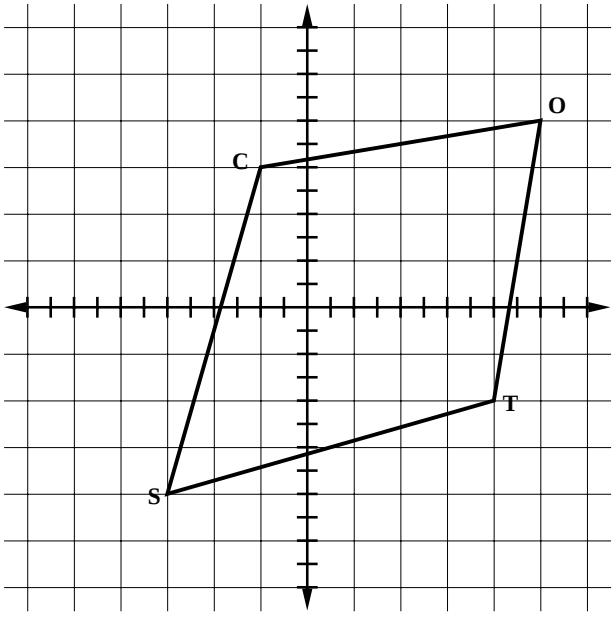
Simplify Irrational Expressions

3.01 Apply algebraic concepts to confirm properties of geometric figures in the coordinate plane.

A. $\overleftrightarrow{BC}$ and $\overline{DE}$ are parallel. Find the perimeters of ABC and BCED.

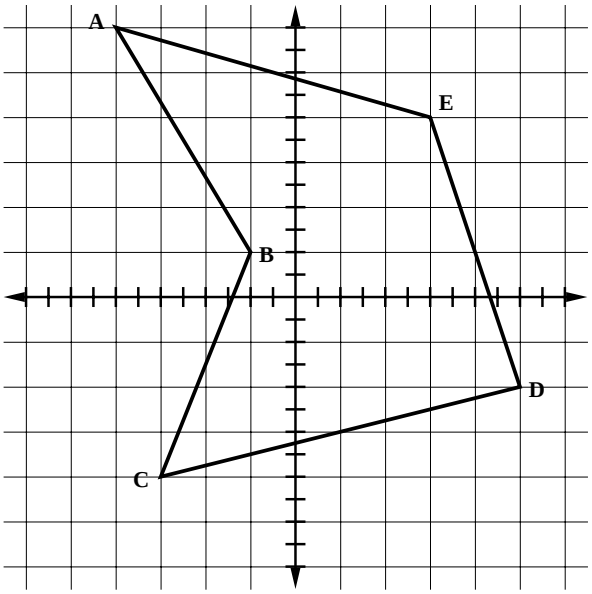


B. Which quadrilateral is TOCS? Justify.

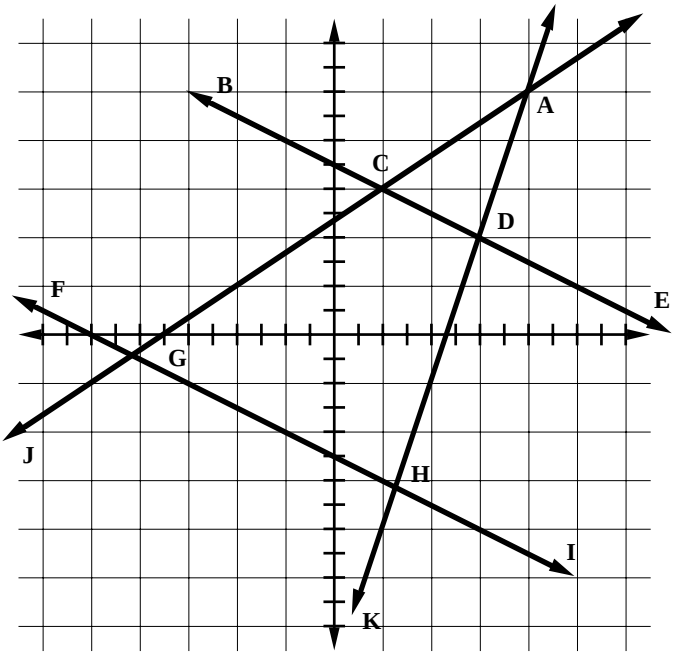


Vocabulary
Concepts
Skills

C. Find the exact area and perimeter of ABCDE.



D. Find m∠FGJ, m∠KHI, and m∠CAD to the nearest hundredth.



Vocabulary
Concepts
Skills

(Multiples of) 90° Rotations

Center of Rotation

Center of Dilation

Mapping

Isometry

Clockwise

Counterclockwise

Pre-image

Image

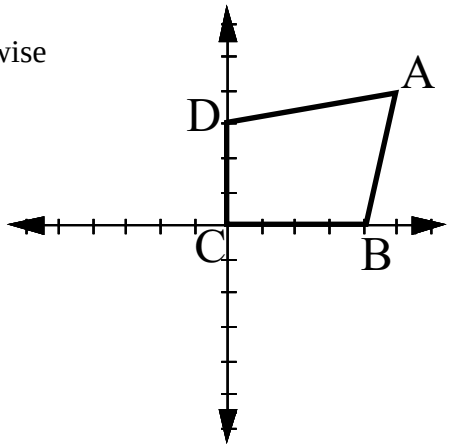
Composition

$(x', y') = (ax + by + c, dx + ey + f)$

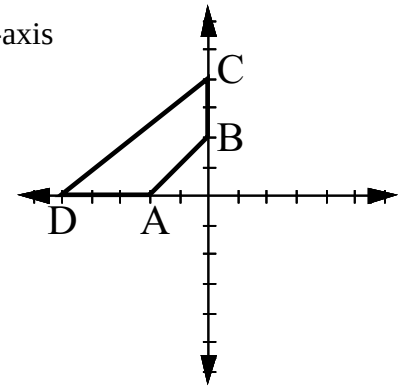
3.02 Describe the transformation (translation, reflection, rotation, dilation) of polygons in the coordinate plane in simple algebraic terms.

A. If $\Delta A'C'B'$ was translated by $(x', y') = (x + 2, y - 6)$ and the coordinates of $\Delta A'C'B'$ are A′(-8, 9), C′(7, -3), and D′(2, 6), what were the coordinates of the pre-image?

B. ABCD is rotated 270° clockwise about the origin. Describe the transformation algebraically.



C. ABCD is reflected across the x-axis and translated five units to the left. Describe the transformation algebraically.



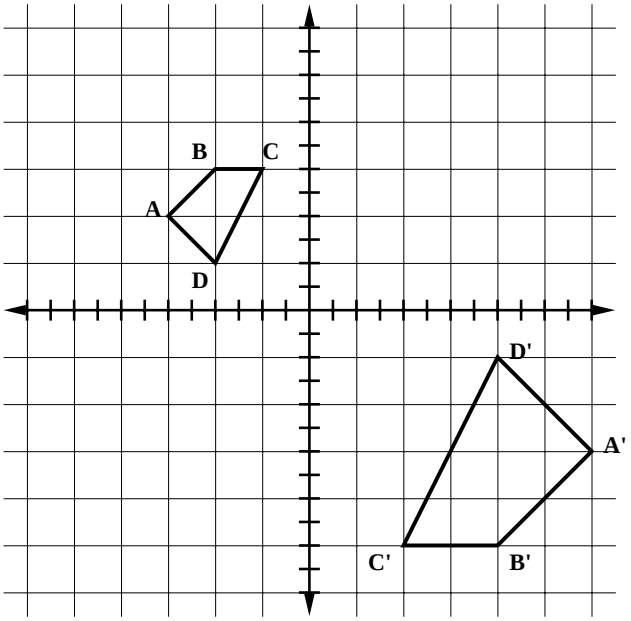
D. ΔABC, with vertices A(2, 8), B(5, 3), and C(6, 8) is transformed according to $(x', y') = (-2x + 3, y - 4)$. Graph ΔABC and $\Delta A'B'C'$; describe the transformation.

E. Transform ΔRST, with vertices R(2, 2), S(3, 6), and T(8, 3), so that its linear dimensions double, but vertex S′ is located at (3, 6). Describe the transformation algebraically.

Vocabulary
Concepts
Skills

F. ΔABC has vertices A(9, 6), B(12, 3), and C(6, -1). ΔPQR has vertices P(1, 6), Q(-2, 3), and R(4, -1). If ΔPQR is the reflected image of ΔABC, what is the equation of the line of reflection? Write the algebaric expression that represents the transformation.

G. Algebraically describe the transformation of ABCD to A′B′C′D′.



Vocabulary
Concepts
Skills

Vertex
Matrix

Standard Matrix Arrangement: alphanumeric left to right and top to bottom

Rows

Columns

Identity Matrix

Unit Matrix

3.03 Use matrix operations (addition, subtraction, multiplication, scalar multiplication) to describe the transformation of polygons in the coordinate plane.

A. $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$ represents ΔABC using its vertices. Transformations of ΔABC are described in each expression. Evaluate each expression and describe ΔA′B′C′ with respect to ΔABC.

(1.) $2\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$

(2.) $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}+\begin{bmatrix} -2 & -2 & -2 \\ 5 & 5 & 5 \end{bmatrix}$

(3.) $\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}+\begin{bmatrix} -3 & 0 \\ 0 & 1.5 \end{bmatrix}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(4.) $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}\cdot\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$

(5.) $\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\cdot\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}$

(6.) $\begin{bmatrix} 1.5 & 0 \\ 0 & 2.5 \end{bmatrix}\cdot\begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{bmatrix}+\begin{bmatrix} -4 & 0 \\ 0 & 7 \end{bmatrix}\cdot\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

B. The coordinates for ΔMAT are M(-6, 8), A(3, 5), and T(-1, -4). Write a matrix expression that would rotate ΔMAT 90° counterclockwise.

C. The coordinates for quadrilateral MNOP are M(-1, 3), N(-5, -1), O(-1, -2), and P(3, 2). Write a matrix expression that will shift MNOP six units left and four units down. What are the coordinates for O′? MNOP is dilated by a facor of 1.4. Write the matrix that represents M′N′O′P′.

Vocabulary
Concepts
Skills

D. What is the matrix expression that will reflect ΔABC, $\begin{bmatrix} 2 & -3 & 1 \\ 4 & 5 & -5 \end{bmatrix}$, over the x-axis? What is the matrix expression that will reflect ΔABC over the y-axis and locate B′ at (0, 0)? Write the matrix expression that dilates ΔABC horizontally by a factor of four and vertically by a factor of three. Write the matrix expression that dilates ΔABC by a factor of two and locates A′ at (9, 3).

E. Write the matrix expression that translates ΔABC, $\begin{bmatrix} 4 & -7 & 2 \\ 1 & 1 & -1 \end{bmatrix}$, so that B′ is at (-6, -2).