Updated 09/15/04

Pre-Calculus

Pre-Calculus provides students a honors-level study of trigonometry, advanced functions, analytic geometry, and data analysis in preparation for calculus. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- ***Describe graphically, algebraically and verbally phenomena as functions; identify independent and dependent quantities, domain, and range, input/output, mapping.***
- ***Translate among graphic, algebraic, numeric, and verbal representations of relations.***
- ***Define and use linear, quadratic, cubic, exponential, rational, absolute value, and radical functions to model and solve problems.***
- ***Use systems of two or more equations or inequalities to solve problems.***
- ***Operate with matrices to model and solve problems.***
- ***Use the trigonometric ratios to model and solve problems.***
- ***Use logic and deductive reasoning to draw conclusions and solve problems.***

Updated 09/15/04

Pre-Calculus

GOAL 1: The learner will describe geometric figures in the coordinate plane algebraically.

1.01 Transform relations in two dimensions; describe the results algebraically and geometrically.
1.02 Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.
 a) Solve using tables, graphs, and algebraic properties.
 b) Interpret the constants and coefficients in the context of the problem.
1.03 Operate with vectors in two dimensions to model and solve problems.

GOAL 2: The learner will use relations and functions to solve problems.

2.01 Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.
 a) Solve using graphs and algebraic properties.
 b) Interpret the constants, coefficients, and bases in the context of the problem.
2.02 Use trigonometric and inverse trigonometric functions to model and solve problems; justify results.
 a) Solve using graphs and algebraic properties.
 b) Create and identify transformations with respect to period, amplitude, and vertical and horizontal shifts.
 c) Develop and use the law of sines and the law of cosines.
2.03 For sets of data, create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions.
 a) Interpret the constants, coefficients, and bases in the context of the data.
 b) Check models for goodness-of-fit; use the most appropriate model to draw conclusions or make predictions.
2.04 Use the composition and inverse of functions to model and solve problems.
2.05 Use polar equations to model and solve problems.
 a) Solve using graphs and algebraic properties.
 b) Interpret the constants and coefficients in the context of the problem.
2.06 Use parametric equations to model and solve problems.
2.07 Use recursively-defined functions to model and solve problems.
 a) Find the sum of a finite sequence.
 b) Find the sum of an infinite sequence.
 c) Determine whether a given series converges or diverges.
 d) Translate between recursive and explicit representations.
2.08 Explore the limit of a function graphically, numerically, and algebraically.

Introduction: Pre-Calculus Standard Course of Study

Vocabulary Concepts Skills

Effects of a, b, c, and d in $y = af(bx + c) + d$

Coefficients

Translation

Reflection

Dilation

Even/Odd

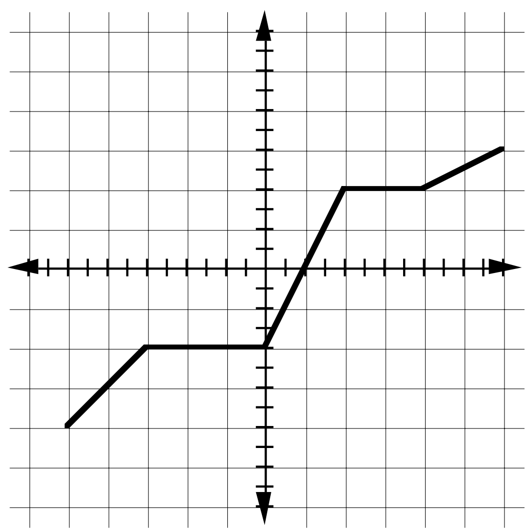
Symmetries

1.01 Transform relations in two dimensions; describe the results algebraically and geometrically.

A. Let $f(x) = x^2 - 4x$. Graph $f(x)$ and $g(x)$. Identify similarities and explain differences between $f(x)$ and $g(x)$.

1. $g(x) = 2f(x)$
2. $g(x) = f(2x)$
3. $g(x) = -f(x)$
4. $g(x) = f(-x)$
5. $g(x) = f(x + 2)$
6. $g(x) = f(x) - 4$
7. $g(x) = |f(x)|$
8. $g(x) = f(|x|)$

B. If the graph of $f(x)$ is given, sketch the graph of $g(x)$.

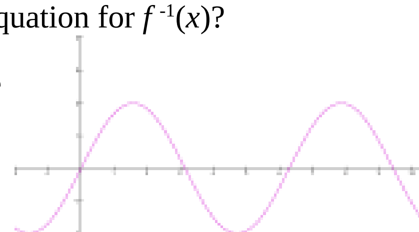


1. $g(x) = -f(x)$
2. $g(x) = f(-x)$
3. $g(x) = |f(x)|$
4. $g(x) = 2f(x)$
5. $g(x) = -0.25f(x)$
6. $g(x) = f(-0.5x)$
7. $g(x) = f(3x)$
8. $g(x) = f^{-1}(x)$
9. $g(x) = f(x - 2) - 3$

C. Plot and label known points on the graph of $g(x)$, using the following clues. $g(x)$ is a polynomial function. $g(x)$ is an odd function. $g(-x)$ contains (3, 0). $g(|x|)$ contains (-4, -14). $g(x - 4)$ contains (5, 4). $-g(x)$ contains (2, -5). Give an equation for $g(x)$.

D. Graph $f(x) = \frac{1}{x}$ and $f(x + 4)$. Compare the domain, range, and asymptotes of the two functions. What are the domain, range, and asymptotes of $f^{-1}(x)$? What is the equation for $f^{-1}(x)$?

E. The figure to the right shows the graph of $y = f(x)$. Sketch $y = |f(x)|$; $y = |f(x)| + 3$; $y = -2 \cdot |f(x)|$.



Vocabulary
Concepts
Skills

Parabola

Circle

Ellipse

Hyperbola

Conic Sections

Standard Form

Center

Focus

Axes

Vertices

Lines of Symmetry

Directrix

Asymptotes

Transformations

Parametric Forms

Solve Equations and Inequalities
Justifying Steps Used

1.02 Use the quadratic relations (parabola, circle, ellipse, hyperbola) to model and solve problems; justify results.

A. According to Kepler's first law of planetary motion, each planet moves in an ellipse with the sun at one focus. Assume that one focus (the Sun) has coordinates (0,0) and the major axis of each planetary ellipse is the x-axis on a cosmic coordinate system (one unit = one billion kilometers). The minimum and maximum distances for Neptune are 4.456 and 4.537 billion kilometers. The minimum and maximum distances for Pluto are 4.425 and 7.375 billion kilometers. For each planet determine the coordinates of the center and second focus; write an equation which represents the orbit; and determine the eccentricity. Graph and describe the orbits.

Vocabulary
Concepts
Skills

Magnitude

Direction

Dot Product

Addition/Subtraction

Scalar Multiplication

Law of Sines

Law of Cosines

1.03 Operate with vectors in two dimensions to model and solve problems.

2.01 Use functions (polynomial, power, rational, exponential, logarithmic, logistic, piecewise-defined, and greatest integer) to model and solve problems; justify results.

Vocabulary Concepts Skills

Independent Variables

Dependent Variables

Domain

Range

Effects of a, b, c, and d in $y = af(bx + c) + d$

Zeros

Intercepts

Asymptotes

Minimum

Maximum

Intersections

End Behavior Models

Increasing/Decreasing

Global versus Local Behavior

Continuous

Discrete

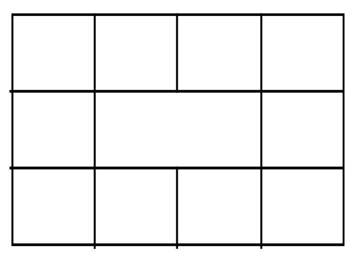
System of Equations

Solve Equations and Inequalities Justifying Steps Used

A. A study showed that the function $M(t) = 6.5\ln(1.4t + 4)$ approximates the population of mice in an abandoned building where t is the number of months since the building was abandoned five years before. Identify the 12-month interval when the mice population grew the most; the least.

B. Each orange tree in a California grove produces 600 oranges per year if no more than 20 trees are planted per acre. For each additional tree planted per acre, the yield per tree decreases by 15 oranges. Describe the orange tree yield algebraically and determine how many trees per acre should be planted to obtain the greatest number of oranges.

C. A real estate developer is planning to build a small office building with a center courtyard. There will be ten rooms, all of the same dimensions, and each rectangular office will have 180 square feet of floor space. The floor plan for the building is shown. All walls are made of cinder block. What dimensions should the rooms have to minimize the total length of the walls to be built? Give answers to the nearest tenth of a foot.



D. Construct tax functions for a single taxpayer (standard deduction, one exemption) using the most recent North Carolina and federal income tax schedules and Social Security (see **B-74** in ***Resources for Algebra***). Graph the functions. Determine the tax liability for gross incomes of \$20,000, \$30,000, and \$100,000. A person's **effective tax rate** is defined as the percent of gross income that is paid in tax. What is the effective tax rate for each of the incomes mentioned? Construct effective tax rate functions based on the income tax schedules and Social Security. Create a composite tax function for a single taxpayer based on the income tax schedules and Social Security. Justify your results.

Vocabulary Concepts Skills

E. The maximum recommended exposure to noise is 85 decibels over a continuous eight-hour period. More exposure risks hearing damage. The maximum exposure is halved for every additional three decibels. Create a maximum noise function and estimate at what noise level hearing damage would occur instantly (in less than one second).

During the 2002 Stanley Cup Finals in Raleigh, the noise level reached 116 decibels in the arena. How long can one stand exposure at that level before damage is sustained?

Vocabulary Concepts Skills

Effects of a, b, c, and d in $y = af(bx + c) + d$

Period

Amplitude

Phase Shift

Frequency

Intercepts

Sinusoidal

Domain/Range

Principle Domain

Asymptotes

Law of Sines

Law of Cosines

Identities

Unit Circle

Radian measure

Degree measure

Sine and Cosine of Special Angles (multiples of π, $\pi/2$, $\pi/3$, $\pi/4$, $\pi/6$)

Solve Equations and Inequalities

Justifying Steps Used

2.02 Use trigonometric and inverse trigonometric functions to model and solve problems; justify results.

A. At a particular location on the Atlantic coast, a pier extends over the water. The height of the water on one of the supports is 5.4 feet, at low tide (2 AM) and 11.8 feet at high tide, 6.2 hours later. Write an equation describing the depth of the water at this location *t* hours after midnight.

Use the form $h(t) = a\cos\left[\frac{2\pi}{T}(t-b)\right] + c$

What will be the depth of the water at this support at 4 AM?

B. Maximum and minimum average daily temperatures of two cities are given. On the same graph, sketch a sinusoidal curve (day of the year, temperature) for each city and create an equation to represent each curve. Explain differences between the curves.

	January 15 (15th day)	July 16 (197th day)
Montreal, Quebec	-10°C	21°C
Orlando, Florida	15°C	28°C

C. Solve log(tan x) = 0.5 for $0 \le x \le 2\pi$.

D. Solve 2sin(0.5x) = 0.75 for $\pi \le x \le 2\pi$.

E. In the interval $[0, 2\pi)$, find the exact solutions for sin 2x - tan x = 0 without a calculator.

F. To find the distance between two points A and B on opposite sides of a lake, a surveyor chooses a point C which is 720 feet from A and 190 feet from B. If the angle at C measures 68°, find the distance from A to B.

G. From the deck of a Cape Fear steamboat, you watch a point on the blade of the paddlewheel as it rotates. The point's distance (*d*) from the surface of the water is a sinusoidal function of time. After three seconds the point on the wheel is at its highest, 16 feet above the surface of the water. The diameter of the wheel is 18 feet, and a complete revolution takes 12 seconds. Sketch a graph of the sinusoidal model. Write an equation for the model. How long does the point remain under water? How far above the surface of the water was the point when the stopwatch read 11 seconds?

Vocabulary
Concepts
Skills

Regression

Residuals

Correlation Coefficient (linear data)

Calculator Limitations (with respect to data)

Translation of Data

Interpret Constants, Coefficients, Bases

Select Best Model

Interpolate

Extrapolate

Estimate

Predict

DRAFT

2.03 For sets of data, create and use calculator-generated models of linear, polynomial, exponential, trigonometric, power, logistic, and logarithmic functions.

A. The data in the table shows the circulation in millions of USA Today from 1985 to 1993. Years are shown as the number of years since 1985.

Years since 1985	0	1	2	3	4	5	6	7	8
Circulation	1.418	1.459	1.586	1.656	1.755	1.843	1.867	1.957	2.001

Use the first and last data points to find an exponential model of the form
$y = ae^{bx}$ for the data. How good is the fit? According to this model, what is the estimated circulation in 2002? Compare the estimate with the actual circulation. What other functions might model the data well? Explain why another function is more reasonable in the context of the problem.

B. Shown are the number of cable television subscribers in the US for several years between 1970 and 1994. Years are expressed as number of years since 1970 and the number of subscriptions is given in millions. Look at a scatter plot of the data and decide on an appropriate function to model the data. The data appears to be leveling off. What in the context explains the leveling off? Looking at your equation model, what is the number to which the data levels off- i.e. what is the carrying capacity?

Number of Years Since 1970	Number of Subscribers (millions)
0	4.5
5	9.8
10	16.0
14	29.0
16	37.5
18	44.0
20	50.0
22	53.0
24	55.3

Vocabulary Concepts Skills

Decomposition to Simpler Forms

$f \circ f^{-1}(x) = x$

$f^{-1} \circ f(x) = x$

Reflection over $y = x$

Domain and Range of Inverses

One-to-One

Domain Restrictions

2.04 Use the composition and inverse of functions to model and solve problems.

A. Scientists are studying the eating and movement patterns of a slow-moving insect. They have found that this insect rarely moves except when seeking food, and then only if it is hungry. If the insect was not very hungry it would not move unless the food was close by. If it was hungry, it would move further. Let d represent the maximum distance it will move in millimeters, and let s represent the degree of satiation (how many centigrams of food is in its stomach). A centigram is one hundredth of a gram. The relation between d and s is given by $d(s) = -1.175s + 73$ if $0 \le s \le 62.4$. Similarly, the scientists have found that the degree of satiation is related to the length of time that has passed since the insect has had a full meal. This relationship is given by $s(t) = 94(2^{\frac{-t}{16}})$ where time, t, is measure in hours. Find a relationship between the times since the insect has eaten its fill and the distance it will move for food, including any domain restrictions. If the insect will travel 50mm for food, how long has it been since it has eaten?

B. Find the inverse of $f(x) = (x - 2)^2 + 3$ if $x \le 2$.

C. Let $f(x) = \dfrac{x}{x-3}$ and $g(x) = \dfrac{x+1}{2x}$. Find and simplify $f(g(x))$ and identify its domain.

Vocabulary Concepts Skills

Polar Coordinate System

Pole

Radius

Magnitude

Direction

Argument

Translate between Rectangular and Polar Coordinates

Graphing Technology

2.05 Use polar equations to model and solve problems.

Vocabulary Concepts Skills

Parameter

Dependent Variables

Independent Variables

Parametric Plots

Motion over Time

Translate between Parametric and Rectangular Forms

DRAFT

2.06 Use parametric equations to model and solve problems.

A. The Sea Queen leaves the port at Nassau at 7:00 PM. She sails due east at 15 mph. The Ocean Princess leaves a small island that is 27 miles north of Nassau at the same time. She sails due south at 22 mph. Assuming the ships continue in the same directions at the same speeds, write parametric equations that model the paths of the two ships as they would appear on a radar screen located at Nassau. Let *d* represent the distance between the two ships. Express *d* as a function of *t*, the number of hours elapsed since 7:00 PM.

Vocabulary Concepts Skills

Linear Sequence

Geometric Sequence

Geometric Series

Subscript Notation

Summation Notation

Converge

Diverge

Translate between Recursive and Explicit Representations

DRAFT

2.07 Use recursively-defined functions to model and solve problems.

A. Mr. Smith has recently retired, and he will rely on his savings to supplement Social Security. He has an account in which the balance was $25,000 on June 1, when he retired. The account earns interest at the rate of 4.8% annual interest, and the interest is compounded monthly. Mr. Smith will withdraw $1100 from the account each month. Write a recursive system to represent the account balance over time. How much money will be in the account 12 months after Mr. Smith retired? After how much time will the balance in the account have dropped to under $5,000?

B. After a person takes pain medication, his kidneys filter the medicine out of his blood stream. During any four hour time period, his kidneys will remove 35% of the medication that remains in the bloodstream. Suppose a patient takes 800 mg of ibuprofen at 8 AM on Tuesday morning. If he does not repeat the dosage, how much medicine will remain his blood stream at 12 midnight on the same day?
Suppose a patient takes 800 mg of ibuprofen at 8 AM on Tuesday morning, and repeats the same dosage every 4 hours. How much medicine will remain his blood stream at 12 midnight on the same day (immediately before he takes his 5^{th} dosage)?
This patient continues taking 800 mg of ibuprofen every 4 hours for several days. The amount of ibuprofen in his bloodstream (in mg) varies between two amounts. What are they?

D. For the sequence of numbers defined as $a_1 = 7$, $a_2 = \sqrt[3]{a_1} + 1$, $a_3 = \sqrt[3]{a_2} + 1$, and in general $a_{n+1} = \sqrt[3]{a_n} + 1$ for all n≥1. What is the value of a_5 to the nearest hundredth?

E. $A_0 = \begin{bmatrix} \mathbf{3} & \mathbf{4} & \mathbf{5} \end{bmatrix}$ and, for n≥1, $A_{n+1} = A_n \bullet \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{2} \\ \mathbf{-2} & \mathbf{-1} & \mathbf{-2} \\ \mathbf{2} & \mathbf{2} & \mathbf{3} \end{bmatrix}$. Use the definition of A_n to generate a sequence of matrices (1≤ n ≤ 6). For the matrices, $A_n = [\mathbf{a_n\ b_n\ c_n}]$, generate a recursive definition for each set of first elements (a_n), second elements (b_n), and third elements (c_n). Write an explicit definition for each set of elements. Explain the relationship between the sets of definitions.

Vocabulary Concepts Skills

Table of Values

Limit

Approaching

Existence

Asymptotes

Continuous

Infinity

End Behavior

2.08 Explore the limit of a function graphically, numerically, and algebraically.

A. Let $v_0 = 4$ and $v_n = \cos(v_{n-1})$. Graphically determine how the value behaves as n approaches infinity. Verify that the limiting value of v_n satisfies the equation $\cos(x) = x$.

B. Let $w_0 = 100$ and $w_n = 0.95w_{n-1} + 80$. Graphicqlly determine how the value of w_n behaves as n approaches infinity. Verify that the limiting value of w_n satisfies the equation $0.95x + 80 = x$.

C. The value of the expression $200\left(1+\frac{0.05}{k}\right)^{10k}$ gives the account balance if $200 is invested for ten years in an account that pays 5% annual interest that is compounded k times each year. Find the account balance if k = 4, 12, 52, 356, 8760, or 525600 (compunded eacch quarter, month, week, day, hour, minute). What value does $200\left(1+\frac{0.05}{k}\right)^{10k}$ approqach as k approaches infinity?

D. Let $f(x) = \frac{x+1}{(x-2)(x+3)}$. Use a table of values to determine what happens to $f(x)$ as $x \to -3$.

E. Describe the behavior of $f(x) = \frac{1}{x}\sin(x)$ for large, positive values of x.

F. Describe the behavior of $f(x) = \sin\left(\frac{\pi}{x}\right)$ as x gets close to zero.