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# Integrated Mathematics 1

*Integrated Mathematics 1* provides students the opportunity to study traditional topics from algebra, geometry, probability, and statistics in a problem-centered, connected approach. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relationships and use those representations to solve problems. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

## Prerequisites

- *Operate with the real numbers to solve problems.*
- *Use formulas to solve problems.*
- *Find, identify, and interpret the slope and intercepts of a linear relation.*
- *Visually determine the line of best fit for a given scatterplot; explain the meaning of the line; and make predictions using the line.*
- *Collect, organize, analyze, and display data to solve problems.*

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**Introduction: Integrated Mathematics 1 Standard Course of Study**

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# Integrated Mathematics 1

**GOAL 1: The learner will perform operations with numbers and expressions to solve problems.**

- 1.01 Write equivalent forms of algebraic expressions to solve problems.
  - a) Apply the laws of exponents.
  - b) Operate with polynomials.
  - c) Factor polynomials.
- 1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.

**GOAL 2: The learner will use properties of geometric figures to solve problems.**

- 2.01 Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.
- 2.02 Develop and apply properties of solids to solve problems.

**GOAL 3: The learner will analyze data and apply probability concepts to solve problems.**

- 3.01 Use graph theory to model relationships and solve problems.
- 3.02 Use theoretical and experimental probability to model and solve problems.
  - a) Use addition and multiplication principles.
  - b) Calculate and apply permutations and combinations.
  - c) Create and use simulations for probability models.
  - d) Find expected values and determine fairness.
- 3.03 Create linear and exponential models, for sets of data, to solve problems.
  - a) Interpret the constants, coefficients, and bases in the context of the data.
  - b) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

**GOAL 4: The learner will use relations and functions to solve problems..**

- 4.01 Use linear functions or inequalities to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants and coefficients in the context of the problem.
- 4.02 Use exponential functions to model and solve problems; justify results.
  - a) Solve using tables, graphs, and algebraic properties.
  - b) Interpret the constants, coefficients, and bases in the context of the problem.
- 4.03 Use systems of linear equations or inequalities in two variables to model problems and solve graphically.

*Vocabulary  
Concepts  
Skills*

Laws  
of  
Exponents

Monomial

Binomial

Trinomial

Polynomial

Addition

Subtraction

Multiplication

Division  
(with monomials)

Factor  
Quadratic  
Expressions

Greatest  
Common  
Factor

Associative  
Properties

Commutative  
Properties

Distributive  
Property

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## 1.01 Write equivalent forms of algebraic expressions to solve problems.

A. Express, in terms of  $x$ , the mean of  $(4x^2 - 6)$ ,  $(2x^2 + 3x)$ , and  $(-13x + 3)$ .

B. Multiply:  $(c - 8)(c + 2)$

C. Multiply:  $(a - 3)(a^2 - 6a - 7)$

D. Simplify:  $x^2(x^2 - 2x) - 3x^2$

E. Simplify:  $(6x^2 - 4x - 3) + (x^2 + x - 5)$

F. Simplify:  $(6x^2y - 3y^4) - (2x^2y - 10)$

G. Simplify:  $(y^2 - 5)(y^3 - 2y^2 - 1)$

H. Simplify:  $(6x^3y^3 - 12x^3y^2 + 8x^6y^5) \div 2xy$

I. In professional baseball, basketball, and hockey, championship series (multiple games between the same opponents) are played to determine the sport's champion. There are two series formats that continue to be used: best three-out-of-five (the first team to win three games is the winner) and best four-out-of-seven (the first team to win four games is the winner). The chance of one team defeating another team in a series can be described by a polynomial where the variable is the first team's record, as a percent, versus the other team for the regular season just completed.

Expand and simplify the polynomial expressions for each series' format.

Three-out-of-five:  $w^3 + 3w^3(1 - w) + 6w^3(1 - w)^2$

Four-out-of-seven:  $w^4 + 4(1 - w)w^4 + 10(1 - w)^2w^4 + 20(1 - w)^3w^4$

J. Factor:  $x^2 - 100$

K. Factor:  $3y^2 + 23y + 14$

L. Factor:  $8x^2y - 24xy - 40x^2y^2$

M. Factor:  $x^3 + 5x^2 + 6x$

N.  $2x^2 + \mathbf{b}x + 15$  factors into the form  $(2x + \mathbf{f})(x + \mathbf{h})$  where  $\mathbf{f}$  and  $\mathbf{h}$  are integers. Find all the values of  $\mathbf{b}$ , if  $\mathbf{b}$  is an integer.

*Vocabulary*  
*Concepts*  
*Skills*

Equation

Model

Formulas

Evaluate

Sequence

Arithmetic  
Sequence

Geometric  
Sequence

$$a_{n+1} = a_n + d$$

$$a_{n+1} = r(a_n)$$

## 1.02 Use algebraic expressions, including iterative and recursive forms, to model and solve problems.

A. The simple interest formula for calculating a bank balance is  $B_{n+1} = (1 + r)B_n$  where  $B_n$  is the beginning balance,  $B_{n+1}$  is the balance at the end of the period, and  $r$  is the interest rate for the period. How much money would be in the account after one year if the annual interest rate is 5% and the principal is \$1000? Suppose you left the balance in the account. What is the new amount after the second year? What would the amount be after the end of the third year? What is the amount after the fourth year?

B. According to Archimedes, “Give me a lever long enough and single-handed I can move the world.” In order to use that lever, a formula is

necessary.  $\frac{W_1}{W_2} = \frac{D_2}{D_1}$  describes the relationship between weights ( $W_1$ ,

$W_2$ ) on a beam and their respective distances ( $D_1$ ,  $D_2$ ) from a fulcrum in order to achieve balance. If you, weighing 145 pounds, need to lift a 400-pound block located 3.5 feet from the fulcrum, how far from the fulcrum must you be?

C. During the investigation of traffic accidents, a piece of evidence that is identified and analyzed is the skid marks left by the cars involved. The formula  $S^2 = 30df$  approximates the relationship among the vehicle’s minimum speed ( $S$ ) in mph, the length ( $d$ ) of the skid marks in feet, and the drag factor ( $f$ ). A recent accident at an intersection showed skid marks for the two cars involved to be lengths of 36' 7" and 42' 7". Find the minimum speeds of both cars. The drag factor for both vehicles was 0.795.

D. Linda was filling out her state income tax return. Her taxable income for the past year was \$37,405. According to the tax rate schedule her tax is \$765 plus 7% of the amount over \$12,750. According to her W-2 form, she paid \$2517.91 in state taxes. Determine the amount Linda owes or should be refunded.

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**Vocabulary**  
**Concepts**  
**Skills**

F. A soup company decides to increase the height of its cans by 40% but to keep the volume the same. Approximately how much must the radius of the can be decreased to keep the volume constant? ( $V = \pi r^2 h$ )

G. Suppose you take 100 units of medication for a cold that is eliminated by your system at a rate of 30% per hour. Assume you do not take additional doses. How many hours will it take for your body to have less than 10 units of medication in it?

H. The NCAA uses the following formula to measure the passing efficiency of quarterbacks.

$$\text{Points} = 8.4 \cdot \frac{\text{yards}}{\text{attempts}} + 100 \cdot \frac{\text{completions}}{\text{attempts}} + 330 \cdot \frac{\text{touchdowns}}{\text{attempts}} - 200 \cdot \frac{\text{intercepts}}{\text{attempts}}$$

These 2003 North Carolina college quarterbacks qualified for the NCAA quarterback passing efficiency ratings. Rank them. (The quarterback with the most points would be considered the most efficient passer.)

	Attempts	Completions	Interceptions	Yards	TD
Durant	389	234	10	2551	18
Gaither	418	246	17	2717	14
Randolph	246	144	10	1773	8
Rivers	438	311	7	4016	29
Robinson	201	133	11	1262	3
Schneider	208	97	6	1220	4
White	157	78	10	1266	12
William	204	120	5	1621	14

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*Vocabulary  
Concepts  
Skills*

Perimeter

Circumference

Lateral  
AreaSurface  
Area

Apothem

Slant  
HeightGreat  
Circle

Height

Altitude

Irregular

Composite

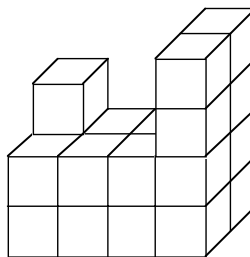
Truncated

Oblique

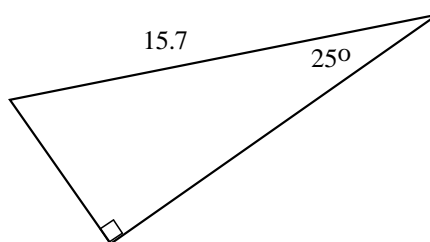
## 2.01 Use the length, area, and volume of geometric figures to solve problems. Include arc length, area of sectors of circles; lateral area, surface area, and volume of three-dimensional figures; and perimeter, area, and volume of composite figures.

A. A cylindrical pipe has an outside radius of 6.5 inches and an inside radius of 5.3 inches. The pipe is six feet long. To the nearest tenth, how much total surface is exposed?

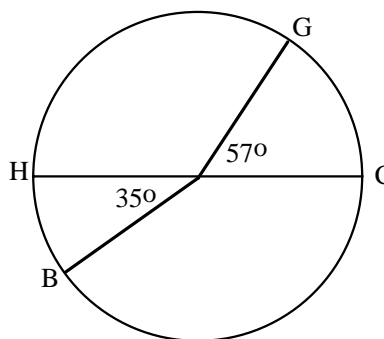
B. The figure shown was built with cubes. The bottom horizontal edge of the figure is 14 cm long. What is the volume of the figure? What is its surface area?



C. Find the perimeter.



D. The circle is divided as shown with diameter  $HC$ , two central angles indicated, and an area of  $414 \text{ in}^2$ . Find the length of arc  $BG$ .



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**Vocabulary**  
**Concepts**  
**Skills**

Sector

Arc

Prism

Pyramid

Cylinder

Cone

Sphere

Cube

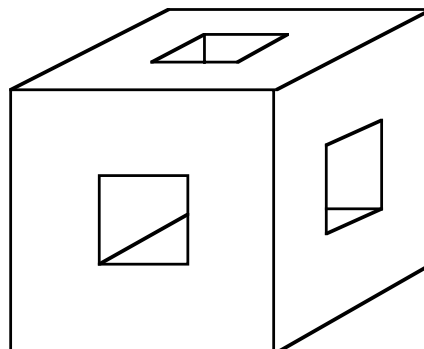
Faces

Vertices

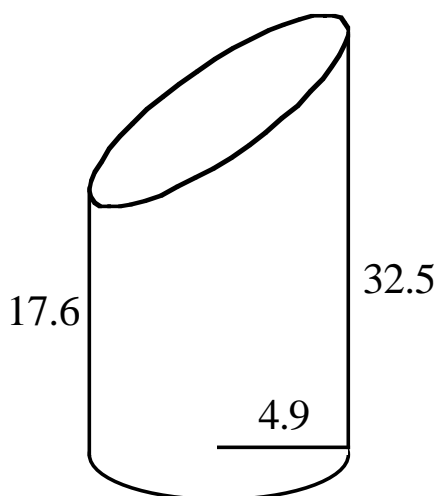
Base

Edge

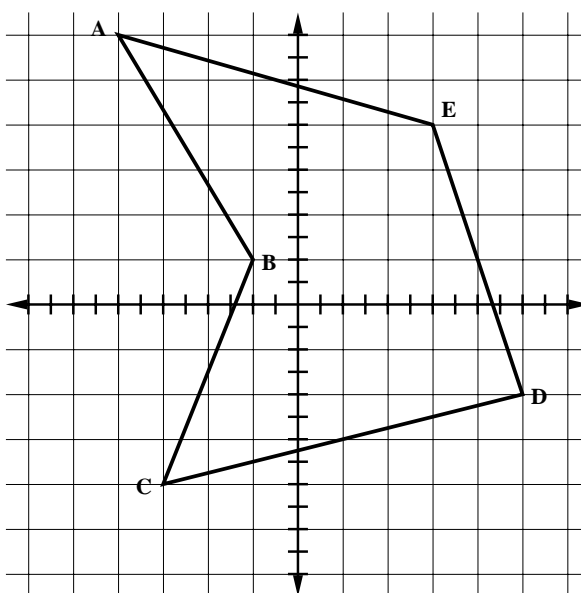
E. The plastic cube shown originally had a volume of  $2500 \text{ cm}^3$ . The front face is drawn to proportion. Square holes were cut through to the opposite face. How much surface is exposed?



F. A glass cylinder is cut as shown. The height goes from 19 to 23 cm and the radius is 5 cm. Find the volume and total surface area. The area of the ellipse is  $\pi Rr$  where  $R$  is the length of the semi-major axis or, in this case, half the length of the diagonal cut.



G. Find the exact area and perimeter of ABCDE.



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*Vocabulary  
Concepts  
Skills*

Edge

Face

Base

Vertices

Cube

Sphere

Cone

Cylinder

Prism

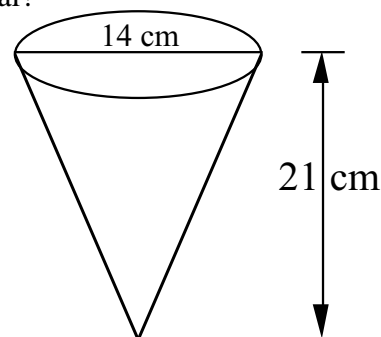
Composite

Truncated

Platonic Solids

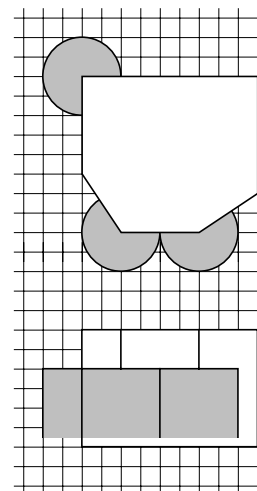
## 2.02 Develop and apply properties of solids to solve problems.

A. If the diameter of the cone shown is increased by 2.5 cm, the volume of the new cone is what percent of the original?

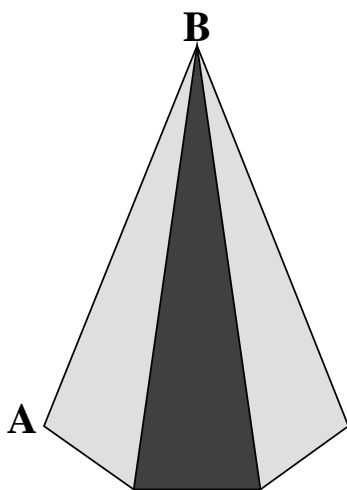


B. A semiregular polyhedron is a solid that has faces in the shape of more than one kind of regular polygon, each vertex is surrounded by the same kinds of polygons in the same order, and each edge is congruent. Construct a truncated cube (a cube with its corners cut off) that is semiregular from a 4 by 4 by 4 cube. What polyhedra is removed from each corner of the cube to form the truncated cube? How long is each edge of the truncated cube? What polygons make the faces of the truncated cube? What is the area of each face of the truncated cube? What is the surface area of the truncated cube? What is the volume of the truncated cube?

C. The top and side views of the new museum are shown. On the grid, 1 unit = 3 m. How much space will be heated or cooled during the year?



D. The base of the pyramid shown is a regular hexagon width of length 13. If point O is the center of the base and OB is 15, what is the measure of  $\angle AOB$ ?



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*Vocabulary*  
*Concepts*  
*Skills*

## **3.01 Use graph theory to model relationships and solve problems.**

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## **3.02 Use theoretical and experimental probability to model and solve problems.**

A. Each day two out of three teams are randomly selected to participate in a game. What is the probability that team A is selected on at least two of the next three days?

B. The local observatory said that a meteor shower could be observed tonight. According to one astronomer, there is a 90% chance of seeing a shooting star between 10:00 and 11:00 PM. Given that the probability of seeing a shooting star is constant throughout the hour, what is the probability of seeing shooting star between 10:00 and 10:10 PM?

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Vocabulary  
Concepts  
Skills

### 3.03 Create linear and exponential models, for sets of data, to solve problems.

Slope

Intercepts

Independent

Dependent

Domain

Range

Scatter Plot

Best-Fit Line

Best-Fit Curve

Regression

Correlation  
Coefficient

Residuals

Estimation

Prediction

Interpolation

Extrapolation

A. The first class postal rates (PR) for 1958-2002 are shown along with the federal minimum wage (MW) for the corresponding years. Based on the data provided, is there a strong relationship between wages and rates? If so, estimate, based on the data provided, the first class postal rate when the minimum wage is \$5.65. What other variables may affect postal rates?

Year	MW	PR	Year	MW	PR	Year	MW	PR
1958	1.00	0.04	1975	2.10	0.13	1991	4.25	0.29
1963	1.25	0.05	1978	2.65	0.15	1995	4.25	0.32
1968	1.60	0.06	1981	3.35	0.18	1999	5.15	0.33
1971	1.60	0.08	1985	3.35	0.22	2001	5.15	0.34
1974	2.00	0.10	1988	3.35	0.25	2002	5.15	0.37

B. The 1998 movie *Titanic* was one of the top money-earning movies of all time. Create a linear model using the data provided. Define the slope in terms of the information provided. When would you expect sales to have reached the \$500 million mark? Is it safe to predict that sales will reach the \$1 billion mark? Explain.

Week	Income	Week	Income	Week	Income
1	28.638	5	242.748	9	376.27
2	88.425	6	274.599	10	402.561
3	157.467	7	308.1	11	4236.983
4	197.881	8	337.355	12	449.157

C. From any vertex of a 4-sided polygon, 1 diagonal can be drawn. From any vertex of a 5-sided polygon, 2 diagonals can be drawn. From any vertex of a 6-sided polygon, 3 diagonals can be drawn. From any vertex of a 7-sided polygon, 4 diagonals can be drawn. How many diagonals can be drawn from any vertex of a 20-sided polygon? Create an equation that will generate the number of diagonals for n-sided polygons.

D. Complete “Scoring and Winning” from *Resources for Algebra* (or one of the web versions found at [www.learnnc.org/dpi/instserv.nsf/Category7](http://www.learnnc.org/dpi/instserv.nsf/Category7)). Define **a** and **b** from the best-fit function  $f(x) = ax + b$ .

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**Vocabulary**  
**Concepts**  
**Skills**

E. When hurricane Fran hit North Carolina on the evening of September 5, 1996, over one million homes and businesses were left without power. Repair crews began immediately restoring electrical service. Based on the graph of the data, which equation, linear or exponential, is most likely to generate a good algebraic model of the hurricane recovery? According to the model, describe the recovery effort. If the recovery continued at the same pace, how long until all power was restored (less than 1,000 customers without power)?

Date	Customers Without Power
Sept.6	1,159,000
Sept.7	804,000
Sept.8	515,000
Sept.9	340,500
Sept.10	195,200
Sept.11	136,300
Sept.12	77,000
Sept.13	37,600

F. The winnings for the 2002 Greater Greensboro Classic (golf tournament) are shown. Determine an algebraic model for winnings based on how a player finished. There were 72 players that finished the tournament. According to your model, how does the money change as a player finishes farther behind the winner?

Place	Winnings	Place	Winnings
1	684,000	38	15,960
2	410,400	44	12,160
3	220,400	48	9667.20
5	144,400	53	8841.33
7	118,433.33	56	8512
10	91,200	61	8208
14	64,600	64	8018
19	47,690	66	7904
23	32,028.57	67	7828
30	22,087.50	68	7752

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*Vocabulary  
Concepts  
Skills*

Graph

Slope

Intercepts

Domain

Range

Independent

Dependent

Relation

Function

Increasing

Decreasing

Properties  
of

Equality

Identities

Commutative  
PropertiesAssociative  
PropertiesDistributive  
Property

Solution

$$y - y_1 = a(x - x_1)$$

$$f(x) = ax + b$$

$$Ax + By = C$$

## 4.01 Use linear functions or inequalities to model and solve problems; justify results.

A. In 1980, the average price of a home in Greensboro was \$40,000. By 2002, the average price of a home was \$120,000. Create a linear model based on this data. According to the model, how much does the cost of a home change annually? Estimate the price of a home in 1991; 2003.

B. The cost of renting a cab is \$3.00 plus twenty-five cents per mile. What would a twenty-five miles trip cost? Suppose you were charged \$12.75. How many miles did you ride in the cab?

C. The power company uses two different rates to calculate a monthly power bill. For July-October, the basic customer charge is \$6.75 plus \$0.08485 per kilowatt-hour. For November-June, the basic customer charge is \$6.75 plus \$0.07485 per kilowatt-hour. 3% North Carolina sales tax is added for the final charge. If the May and September bills are both \$127, what is the difference in the amount of power (kilowatt-hours) used each month?

D. For the line  $y = ax + b$  where  $a > 0$  and  $b > 0$ :  
 If  $b$  increases and  $a$  remains constant, how does the  $x$ -intercept change? What happens to the line?  
 If  $a$  increases and  $b$  remains constant, how does the  $x$ -intercept change?  
 If  $b$  is multiplied by  $-1$  and  $a$  remains constant, how does the line change?  
 If  $a$  decreases, getting closer to 0, and  $b$  remains constant what happens to the line?

E. A telephone company offers two long distance calling plans. The alpha plan charges \$0.10 per minute and the beta plan charges \$0.07 per minute plus a monthly charge of \$3.95. For a one-month period, describe when each plan is the least expensive. When are costs equivalent?

*Vocabulary*  
*Concepts*  
*Skills*

Increasing

Decreasing

Coefficient

Base

Intercepts

Initial Value

Exponent

$$f(x) = ab^x + c$$

## 4.02 Use exponential functions to model and solve problems; justify results.

A. In 1998 there were approximately 430,000 people employed in the United States as computer support specialists. According to the US Bureau of Labor Statistics, that number was expected to grow 7% annually. The function  $c(x) = 430000(1.07)^x$  describes the growth of specialists after  $x$  years (since 1998). According to the function, how many computer support specialists will there be in 2003? When will the number reach one million?

B. A sports utility vehicle (SUV) was purchased for \$23,000 recently. According to the automobile sales association, the buyer can expect the SUV to lose about 11% of its value annually.

The function  $v(x) = 23000(0.89)^x$  describes the value of the SUV  $x$  years after its purchase. The SUV is valued at half its original price after how many years? In ten years how much could an owner expect to sell the SUV for?

C. In the United States for the period 1980-1998, the function  $f(x) = 23.5(0.978735)^x$  describes the fatality rate (fatalities per 100,000) due to motor vehicle accidents ( $x = 0$  for 1980). Describe the trend for the period indicated. How is the fatality rate changing annually?

The number of miles traveled increased annually as well as the number of registered motor vehicles for the same period. Are these variables that affect the fatality rate? If not, name several that are.

D. The average weekly food cost for a family of four in 1990 was \$128.30. For the next ten years the weekly food cost increased 2.45% annually. The function  $f(x) = 128.3(1.0245)^x$  represents the cost of food for that period. Estimate the weekly food cost for a family of four in 2003. How much has food cost increased since 1990?

E. Bacteria growing on discarded food triples every five hours. If there are one million bacteria present now, how many will there be one day later?

F. At the end of four years ( $t$ ), a savings account paying 5.35% annually ( $r$ ) compounded continuously, had a balance ( $B$ ) of \$3096.56. What was the initial deposit ( $P$ )? (Use  $B = Pe^{rt}$ ) If the initial deposit had been in an account compounded annually, how much less interest would have been earned?

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**Vocabulary**  
**Concepts**  
**Skills**

G. Over the last year the stock value of an internet company has dropped at a rate of 17% per month. The value of the stock at the beginning of the year was \$19.50. What was the value of the stock at the end of the year? If the stock's value continues to decrease at the same rate, how long does it take the stock to be worth one-tenth of its original value?

H. For  $f(x) = 5 \cdot b^x$  and  $b > 1$ : As **b** increases, how does the graph of  $f(x)$  change?

I. For  $f(x) = 5 \cdot b^x$  and  $0 < b < 1$ : As **b** approaches zero, how does the graph of  $f(x)$  change?

J. For  $f(x) = a \cdot 1.9^x$  and  $a \geq 1$ : As **a** increases, how does the graph of  $f(x)$  change?

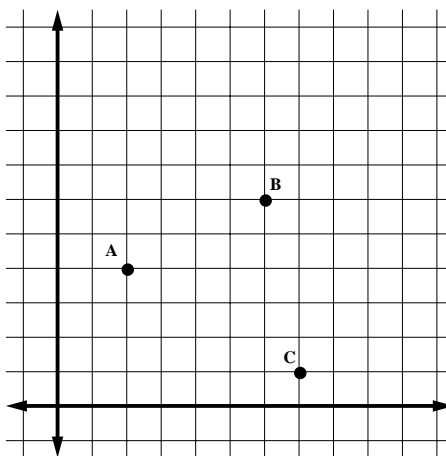
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## 4.03 Use systems of linear equations or inequalities in two variables to model problems and solve graphically.

A. Transportation costs for travel between selected cities are shown below. Assume that the costs identified represent a linear trend. Determine the distance at which driving a car is less expensive than riding the train. When does it become cheaper to fly rather than drive? For what distance is the train the most expensive mode of travel? Identify some advantages and disadvantages for each mode of transportation. What other variables affect the cost of travel?

	Distance	Car	Air	Rail
Raleigh – Charlotte	300	\$108	\$224	\$40
Raleigh – New York	1300	\$468	\$169	\$147

B. A, B, and C are three vertices of a parallelogram. What are the coordinates of the fourth vertex of the parallelogram? Identify all the solutions.



C. Mildred makes \$4 an hour babysitting and \$7 an hour when she works at a local fast-food restaurant. Her parents do not want her to work more than 20 hours per week. Mildred would like to earn at least \$70 a week. Write a system of inequalities that show the number of hours she could work at each job. Graph the system. Identify at least four possible solutions.

D. The Twin Theater charges \$7.50 for adult tickets and \$4 for children 12 or under. The theater has 470 seats. The manager wants to have a nightly income (two shows) of at least \$5000. Write a system of inequalities for the number of children and adult tickets that can be sold. Identify at least four possible solutions.

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