

Updated 03/01/05

Discrete Mathematics

Discrete Mathematics introduces students to the mathematics of networks, social choice, and decision making. The course extends students' application of matrix arithmetic and probability. Applications and modeling are central to this course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.

Prerequisites

- *Describe graphically, algebraically and verbally phenomena as functions; identify independent and dependent quantities, domain, and range, input/output, mapping.*
- *Translate among graphic, algebraic, numeric, and verbal representations of relations.*
- *Define and use linear and exponential functions to model and solve problems.*
- *Operate with matrices to model and solve problems.*
- *Define complex numbers and perform basic operations with them.*

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Introduction: Discrete Mathematics Standard Course of Study

Updated 03/01/05

Discrete Mathematics

GOAL 1: The learner will use matrices and graphs to model relationships and solve problems.

- 1.01 Use matrices to model and solve problems.
 - a) Display and interpret data.
 - b) Write and evaluate matrix expressions to solve problems.
- 1.02 Use graph theory to model relationships and solve problems.

GOAL 2: The learner will analyze data and apply probability concepts to solve problems.

- 2.01 Describe data to solve problems.
 - a) Apply and compare methods of data collection.
 - b) Apply statistical principles and methods in sample surveys.
 - c) Determine measures of central tendency and spread.
 - d) Recognize, define, and use the normal distribution curve.
 - e) Interpret graphical displays of data.
 - f) Compare distributions of data.
- 2.02 Use theoretical and experimental probability to model and solve problems.
 - a) Use addition and multiplication principles.
 - b) Calculate and apply permutations and combinations.
 - c) Create and use simulations for probability models.
 - d) Find expected values and determine fairness.
 - e) Identify and use discrete random variables to solve problems.
 - f) Apply the Binomial Theorem.
- 2.03 Model and solve problems involving fair outcomes:
 - a) Apportionment.
 - b) Election Theory.
 - c) Voting Power.
 - d) Fair Division.

GOAL 3: The learner will describe and use recursively-defined relationships to solve problems.

- 3.01 Use recursion to model and solve problems.
 - a) Find the sum of a finite sequence.
 - b) Find the sum of an infinite sequence.
 - c) Determine whether a given series converges or diverges.
 - d) Write explicit definitions using iterative processes, including finite differences and arithmetic and geometric formulas.
 - e) Verify an explicit definition with inductive proof.

*Vocabulary
Concepts
Skills*

Row

Column

Dimensions

Inverse

Identity

Scalar

Transpose

Vectors

Matrix
Operations

Vectors

Leslie Model

Markov Chain

Leontief
Input/Output
Model

Cryptography

Cipher

Code

1.01 Use matrices to model and solve problems.

A. A breed of lizard has a life span of at most three years. A female lizard will usually produce her first offspring at six months of age and continue producing offspring until she dies. Only 20% of newborn lizards reach the age of six months. Of those who do, 70% reach the age of one year. Of the ones who reach one year, 90% reach the age of 18 months. Of those, 80% reach the age of two years and 50% of those reach two and one-half years. None of the lizards live past the age of three years. The birth rates by ages in months are as follows: 0 to 6 - 0; 6 to 12 - 1.5; 12 to 18 - 1.9; 18 to 24 - 2.6; 24 to 30 - 3.1; and 30 to 36 - 1.4.

Let $P_0 = [100 \ 0 \ 0 \ 15 \ 0 \ 10]$.

Find the population distribution after four years have passed (this is P_8) and after seven years have passed. Also find the total population at each of these times.

B. A video store owner has found that the probability that a customer who rented a movie today will also rent a movie tomorrow is 35% while the probability that a customer who did not rent today, will rent tomorrow is 10%. Write the transition matrix that represents this information. If 853 out of his 8745 customers rented a movie on Monday night (7892 of his customers didn't rent one on Monday), about how many customers can he expect to rent a movie on Tuesday? About how many of his customers can he expect to rent a movie three weeks from Monday?

C. Suppose that in the final days of a political campaign the Democrats and Republicans are planning their strategies for winning undecided voters to their political camps. The Democrats have decided on two strategies, plan A and plan B and the Republicans plan to counter with plan C and plan D. The numbers in the matrix represent the percentage of undecided voters joining the Republicans in each case. Determine the best strategy (or combination of strategies) for each party and the expected outcome if both parties use their best strategies.

	Plan C	Plan D
Plan A	70	40
Plan B	50	60

D. Encode POND using matrices. The coding matrix is $\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

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E. Mrs. Ormond notices that the students' test performance is affected by how well or how poorly they did on the last test. Specifically, 75% who did well on the last test do well on the next test, 20% do average, and 5% do poorly. Of the average scorers on the last test, 70% score average on the next test, 10% do well, and 20% do poorly. For those who did poorly on the last test, 86% do poorly on the last test, 10% do average, and 4% do well. If Billy did average on the first test, how likely is he to do well on the fourth test? (that's three after the first one.)

F. Decode SBEICSTE using matrices. It was encoded using $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

G. A population of laboratory animals model the following birth and survival rates.

Age	Birthrate	Survival Rate
0 - 3	0.3	0.9
3 - 6	0.9	0.7
6 - 9	0.9	0.8
9 - 12	0.6	0

If the initial population is

Months	0 - 3	3 - 6	6 - 9	9 - 12
Number	10	11	8	4

How many newborn animals were there after four cycles? What is the total population after two cycles? When will the population reach 800? What is the long term growth rate?

H. Joe's Motors and Bob's Batteries make up a two sector economy. For the motor sector to produce 100 motors, it must use 2% of its own motors. For every 100 motors produced, three batteries (3% of the motors produced) are required by Bob's Batteries. For the battery sector to produce 100 batteries, it must use 4% of its own batteries. For every 100 batteries produced, two motors (2% of the number of batteries produced) is required from Joe's Motors.

Joe's Motors has an order for 5500 motors and Bob's Batteries has an order for 6000 batteries. Draw a weighted digraph that summarizes production needs. What is the internal production needed for each company? Find the total production needed.

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1.02 Use graph theory to model relationships and solve problems.

*Vocabulary
Concepts
Skills*

- Edges
- Vertices
- Degree
- Paths
- Circuits
- Cycle
- Isomorphic
- Homomorphic
- Connected/
Complete
- Trees
- Digraphs
- Adjacent
- Multigraphs, Loops
- Minimum
Spanning Tree
- Euler
Circuits and Paths
- Hamiltonian
Circuits and Paths

A. In scheduling committee meetings for school improvement, six different meetings need to be scheduled. Based upon the information shown about which faculty members need to attend which meetings, what is the fewest number of time slots which could be used to schedule these six meetings? Which meeting could be scheduled with the Music/Art meeting?

Committee	Adam	Betty	Carl	Don	Eddy	Frank	Gus	Hank	Inez
Calendar	X			X				X	X
Academics	X					X	X	X	
Sports		X				X			X
Music / Art		X	X						X
Neighbors		X		X	X		X	X	
Building			X		X	X	X		

B. Use the task list at right. Find the earliest start time for each task. Determine the minimum project time. List the critical path. Find the latest start time for task C.

Task	Time	Pre - requisites
A	4	none
B	6	none
C	9	A
D	7	A,B
E	5	B
F	6	D
G	2	E,H
H	3	C,F
I	7	F
Finish		G,I

C. Represent $(7 - 2) \cdot 6 + 3 \cdot (14 / 7)$ as a binary expression tree.

D. Examine the complete bipartite graph $K_{4,5}$. How could you best describe the complement of that graph? Is the complement connected or not? Describe, as precisely as possible, the complement of the graph $K_{m,n}$.

E. Draw a tree with four vertices in which two vertices have a degree of three, one has a degree of two, and one has a degree of four.

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Vocabulary
Concepts
Skills

(continued)

Bipartite
 Graphs

Chromatic Number
 of a Graph

Kruskal's
 Algorithm

Prim's
 Algorithm
 Optimization

Routing
 Problems

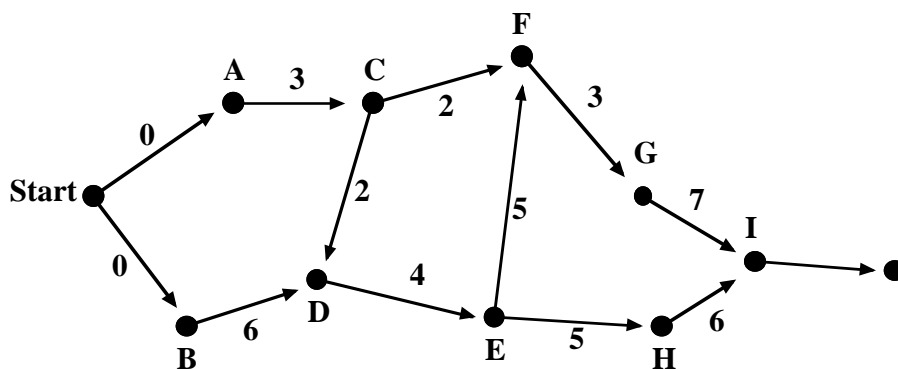
Traveling
 Salesman
 Problems

Critical
 Paths

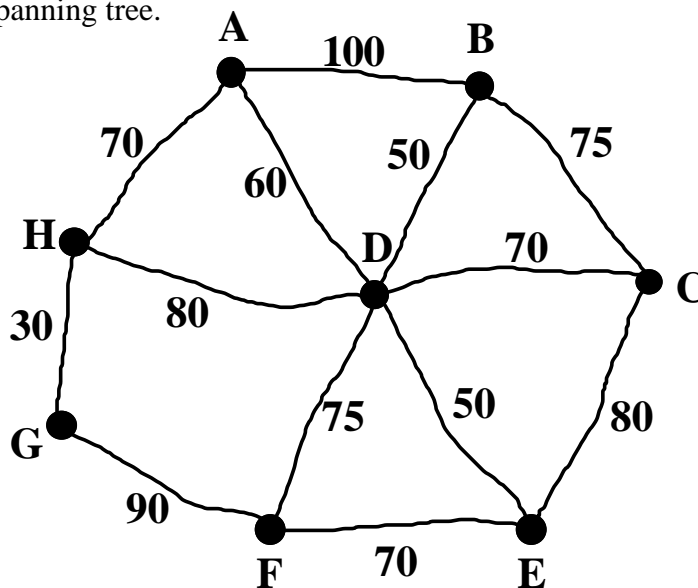
Earliest/Latest

Minimization

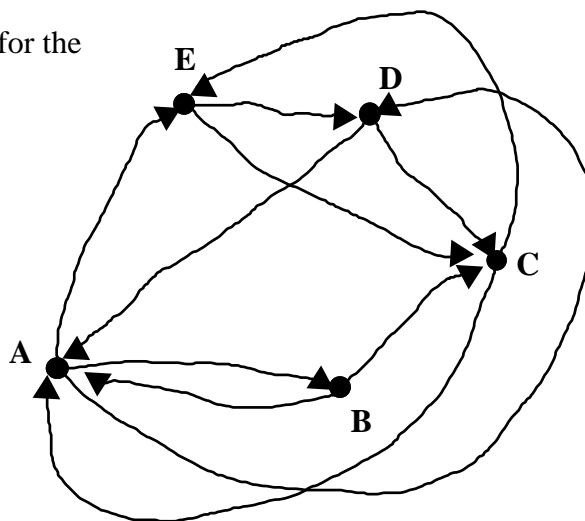
F. According to the graph shown, how many prerequisites does F have? What is the latest start time for E? What is the minimum project time? What is the critical path?



G. Use Kruskal's algorithm to find the minimum spanning tree.



H. Find an Euler circuit for the directed graph.



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Vocabulary
Concepts
Skills

Mean

Median

Variance

Standard
Deviation

Normal
Distribution

Random
Sampling

Census

Survey

Bias

Population

Venn Diagrams

2.01 Describe data to solve problems.

A. The averages of 28 students in a Geometry class are given below:

82, 100, 94, 68, 34, 72, 70, 96, 99, 92, 90, 85, 70, 46, 71, 71, 77, 78, 95, 82, 80, 100, 99, 72, 69, 74, 84, 87

Find the median, mean, and standard deviation. Make a normal distribution curve.

B. 125 people have decided to take a trip. 46 people said they would be happy either going to New York City or Hashville. 38 people would be happy with Nashville or Orlando and 27 people would be happy with New York City or Orlando. 14 people only want to go to New York City, 8 people only to Nashville, and 12 people only to Orlando. How many said they would be happy with any of the three places? How many would be happy if Orlando were chosen?

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*Vocabulary
Concepts
Skills*

2.02 Use theoretical and experimental probability to model and solve problems.

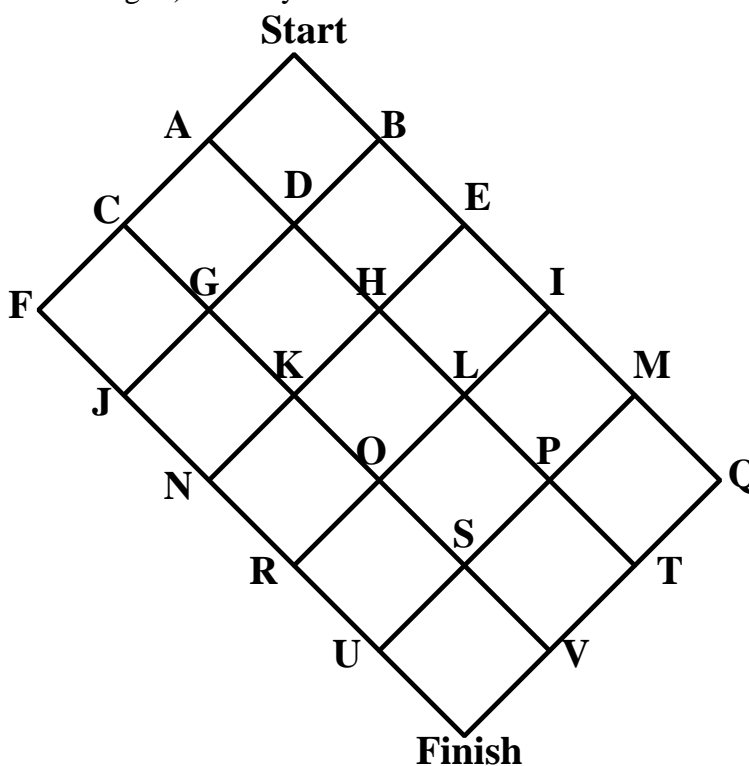
- Counting
- Combination
- Permutation
- Factorial
- Event
- Success
- Trial
- Sample Space
- Mutually Exclusive
- Disjoint
- Dependent/Independent
- Binomial Probability
- Expected Value
- Random Variable
- Game Theory
- Simulation
- Monte Carlo

A. A scopa deck consists of 40 cards in four suits (clubs, cups, coins and swords) numbered from one to seven with a page (8 points) a knight (9 points) and a king (10 points). How many different sets of cards could be picked up with the five of coins? (The point total must equal five.)

Plays that pick up 1 card	3
Plays that pick up 2 cards	_____
Plays that pick up 3 cards	_____
Plays that pick up 4 cards	_____
Total plays	_____

B. How many different positive sums of money can be made using a 25¢ coin, \$1, \$5, \$10, \$20 and \$50 bills? (One of these sums of money is \$1.25, another is \$31.)

C. How many alphabetical paths exist from Start down to Finish in the graph shown? An alphabetical path is one in which the vertices are visited in an alphabetical order. Start-A-D-H-K-O-S-V-Finish is alphabetical but Start-A-D-H-K-O-L-P-S-V-Finish is NOT alphabetical. (Hint: Each path consists of 8 edges that can be classified into either down/left or down/right.) Show your work/formula.



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Vocabulary
Concepts
Skills

D. In the U.S. Senate there are 100 senators. It takes a $\frac{2}{3}$ majority to override a presidential veto. This means that a coalition of at least 67 senators is required to override. How many different minimum coalitions exist that could override a veto? (A minimum coalition is one in which the loss of even one vote would cause the coalition to be insufficient.)

E. A game of chance at the State fair lets a person choose a number between one and six and then roll two dice. They must pay one dollar to play. They win one dollar if their number appears on one die and two dollars if their number appears on two dice. They lose one dollar if their number doesn't appear at all. If you play the game 100 times, how much money should you expect to win or loss?

F. Roll a die. name two possible events that are not mutually exclusive. name two possible events that are mutually exclusive.

G. Find the best strategies for thre row and column players given the payoff matrix below. What is the saddle point, if any?

-2	3	0
4	-1	-3
2	4	-1

H. Determine how many ways a coach can choose the starting players on his football team. He has three quarterbacks available; he will select three receivers from the ten available; he will select one running back from the four available; and he will select six linemen from the remaining 33 players.

I. Each day two out of three teams are randomly selected to participate in a game. What is the probability that team A is selected on at least two of the next three days?

J. The local observatory said that a meteor shower could be observed tonight. According to one astronomer, there is a 90% chance of seeing a shooting star between 10:00 and 11:00 PM. Given that the probability of seeing a shooting star is constant throughout the hour, what is the probability of seeing shooting star between 10:00 and 10:10 PM?

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*Vocabulary
Concepts
Skills*

Hamilton

Jefferson

Adams

Webster

Hill

Quota
(Lower/Upper)

Violation of Quota
(Quota Rule)

Balinski and Young
Impossibility
Theorem

Alabama Paradox

Population Paradox

New States
Paradox

2.03 Model and solve problems involving fair outcomes.

a) solve problems involving apportionment

A. A very small country has five states and needs to apportion their legislature of 93 members according to their respective populations.

Aram	5576
Bethel	1387
Capernaum	3334
Damascus	7513
Elam	311
Total	18121

Apportion the 93 representatives according to the methods of Hamilton, Jefferson, Adams, Webster and Hill.

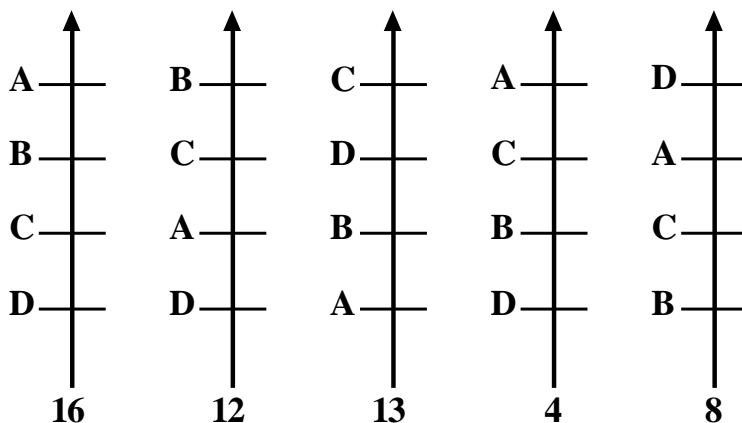
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2.03 Model and solve problems involving fair outcomes.

b) solve problems involving election theory

- Vocabulary
- Concepts
- Skills
- Plurality
- Majority
- Borda Count
- Runoff
(plurality with runoff)
- Sequential
Runoff
(plurality with
elimination)
- Condorcet
- Approval
Voting
- Arrow's
Conditions
- Majority Criterion
- Condorcet Criterion
- Monotonicity
Criterion
- Independence of
Irrelevant
Alternatives Criterion

A. Use the preference schedules above to find the Borda count winner.
If B decides to withdraw, who will be the plurality winner?



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Vocabulary
Concepts
Skills

Weighted
Voting

Banzhaf
Power
Index

Shapley/Shulik
Power
Index

2.03 Model and solve problems involving fair outcomes.

c) solve problems involving voting power

A. Find the Shapley-Shubik power index for each voter.

$$[q: A, B, C] = [10: 3, 7, 9]$$

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*Vocabulary
Concepts
Skills*

Desired
Outcome

Perceived
Value
(Fair Share)

Divider/Chooser

Lone
Divider

Last
Diminisher

Marker
Method

Estate
Division
(Sealed Bids)

2.03 Model and solve problems involving fair outcomes.

d) solve problems involving fair division

A. Ann, Bob, Carol, and Don are the four heirs of an estate that includes a small house, a cabin in the mountains, a 1965 mustang and \$95,000 in cash. They decide to divide the estate using sealed bids. Their bids on the three main items are listed in the following table. Each one is entitled to one-fourth of the estate.

Bids on:	Ann	Bob	Carol	Don
House	66,000	70,000	72,000	68,000
Cabin	42,000	37,000	36,000	40,000
Mustang	6,000	6,500	4,000	5,200

What does Ann receive as her share of the estate?

What does Bob receive as his share of the estate?

What does Carol receive as her share of the estate?

What does Don receive as his share of the estate?

Observe the situation from Bob's perspective. What is the dollar value of the share of each heir from Bob's view?

B. Aunt Marge died recently and left a 1965 Porche, a diamond necklace, a condominium, and \$10,500 in cash to her three nieces, Donna, Toni, and Linda. They decided to bid on each item. Find the final settlement for each person.

	Donna	Linda	Toni
Porche	3,000	2,800	5,000
Necklace	800	3,000	2,100
Condo	80,000	83,000	79,000

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*Vocabulary
Concepts
Skills*

Fibonacci
Sequence

Golden
Ratio

Arithmetic
Sequences

Geometric
Sequences

Close Form

Fractal

Cobweb

Growth and Decay

Sequences

Series

Inductive Proof

3.01 Use recursion to model and solve problems.

A. For the following sequence: 5.3, 9, 12.7, 16.4, 20.1, 23.8, ... Determine whether the sequence is arithmetic, geometric, mixed or something else. Determine a recursive definition that will generate the sequence. Determine a closed form function that will generate the sequence. Find the value of the tenth term in the sequence.

B. A recurrence relation is described by the equation $t_n = 3t_{n-1} - 7$. What is the “fixed point” for this relation?

C. Find the sum of the infinite geometric series shown:
 $54 + 18 + 6 + 2 + \dots$

D. Sarah puts \$300 in a savings account that earns 5.25% compounded annually. Write a recursive relation for the situation. When will Sarah double her money?

E. Find the 20th term of the sequence 10, 15, 22.5, 33.75, 50.625, ...

F. An auditorium has 18 seats in the first row. Each successive row has two additional seats. The last row has 84 seats. Write a recursive relation for the number of seats in the n^{th} row. Write a closed form solution for the number of seats in the n^{th} row. How many rows are there?

G. Write a recursive relation for the number of edges (E_n) in a complete graph with n vertices.

H. $A_0 = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$ and, for $n \geq 1$, $A_{n+1} = A_n \cdot \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix}$. Use the

definition of A_n to generate a sequence of matrices ($1 \leq n \leq 6$). For the matrices, $A_n = \begin{bmatrix} a_n & b_n & c_n \end{bmatrix}$, generate a recursive definition for each set of first elements (a_n), second elements (b_n), and third elements (c_n). Write an explicit definition for each set of elements. Explain the relationship between the sets of definitions.

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