The learner will demonstrate an understanding of patterns, relationships, and elementary algebraic representation.

5

5.01 Describe, extend, and generalize numeric and geometric patterns using tables, graphs, words, and symbols.

Notes and textbook references

A. Have the students investigate patterns on the hundred board. Use markers or chips to cover certain numbers, then ask for the rule and an extension of the pattern. Example: On an overhead hundred board cover the numbers 8, 14, 20, 26, and 32. Ask the students to describe and extend the pattern. The description should include that the number increases by six. This is a growing pattern.

B. Draw patterns on the board with figures or pictures for students to practice analyzing, describing, and extending the patterns numerically. Example: Let the students show their strategies for determining their solutions on the board.

C. Pass out manipulatives to students to make patterns. Each person needs to record their pattern in symbols on paper. Switch papers with a partner to let the partner describe the pattern and extend it to the tenth term. Transfer the pattern to a table to explain to the partner how the tenth term (or any term) can be found.

D. Explore sequences on the calculator by experimenting with the automatic constant function. Have the students key in any number on their calculator, for example, 14. Select any one digit number, for instance 3, and skip count on their calculators and record the sequence as it appears in the display. For example, 14 + 3 = = = . . . Explore patterns with skip counting before moving into growing patterns without constants, like square numbers.

Notes and textbook references

Additional grid paper is available on Blackline Master V - 11.

E. See Blackline Master V - 1. Connect the points. How many line segments can you draw starting with two points? three points? four points? five points? Is there a pattern?

Two points can be connected with one line segment.
Three points can be connected with three line segments.
Four points can be connected with six line segments.
Five points can be connected with ___ line segments.

F. With calculators and hundred boards, students work in pairs. Add the numbers in the first horizontal row (total 55). Continue adding the numbers in the next three horizontal rows (total 155, 255, 355. . .). Analyze totals to see the pattern that develops. Predict the next totals and test your predictions. Discover why this pattern occurs (because each number increases by 10, and ten 10's equal 100). *Extension:* Add vertical columns and follow the same procedure.

G. List the first 15 or more multiples of 12, beginning with zero: 0, 12, 24, 36, 48 . . . Is there a pattern? In which place is the pattern consistent? (one's place: 0, 2, 4, 6, 8, 0, 2, 4 . . .). Look for similar patterns in other multiples. Writing the multiples vertically helps.

H. Three people are introduced and shake hands with each other exactly once. How many handshakes are exchanged altogether? Person A shakes hands with persons B and C. Person B shakes hands with Person C. Three handshakes are exchanged. What happens when four persons shake hands? five persons? six persons? ten persons? How could you figure the handshakes for 50 people or 100 people or any number of people?

Notes and textbook references

I. Find the numbers on a hundred board which are palindromes. Color these. Next, search for numbers which will become palindromes in one step. Color these a different color. Examine all numbers on the hundreds board and color them according to the number of steps it takes to make a palindrome. In each step the number (or resulting sum) is added to the digits written backwards [i.e., 17 + 71 = 88 (one step); 28 + 82 = 110, 110 + 011 = 121 (two steps)]. What patterns do you see on the hundred board?

One-Step Palindrome

$$\begin{array}{r} 24 \\ +\ 42 \\ \hline 66 \end{array}$$

Two-Step Palindrome

$$\begin{array}{r} 57 \\ +\ 75 \\ \hline 132 \end{array} \qquad \begin{array}{r} 132 \\ +\ 231 \\ \hline 363 \end{array}$$

Three-Step Palindrome

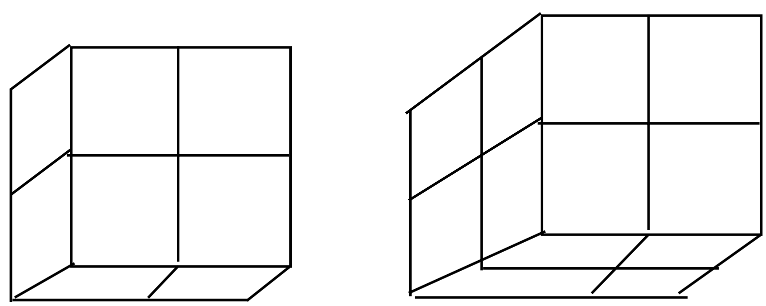
$$\begin{array}{r} 86 \\ +\ 68 \\ \hline 154 \end{array} \qquad \begin{array}{r} 154 \\ +\ 451 \\ \hline 605 \end{array} \qquad \begin{array}{r} 605 \\ +\ 506 \\ \hline 1111 \end{array}$$

Notes and textbook references

J. Use a calculator. What patterns do you see?

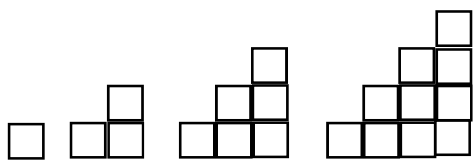
3 x 7 x 8 x 1 x 13 x 37 =_____.
3 x 7 x 6 x 2 x 13 x 37 =_____.
3 x 7 x 5 x 3 x 13 x 37 =_____.
3 x 7 x 9 x 4 x 13 x 37 =_____.
3 x 7 x 4 x 5 x 13 x 37 =_____.

K. A frame with one layer takes 4 cubes, 2 layers takes 8 cubes.



Make a table to show frames with 3 to 10 layers. How many cubes for 30 layers?
Suppose you started with a frame whose first layer took 9 cubes. Two layers would take 18. What would the table look like if there were 3 to 10 layers? 30 layers?

Extension: Make a table to show the number of blocks used to build up 10 steps. How many cubes for 100 steps? What if you used toothpicks?



L. On a calendar mark off a 4 x 4 square. Add the diagonals. What happens? Try this on at least two other 4 x 4 squares. Write about what you notice. Try it with some 3 x 3 squares. What do you observe? Compare the answers with these 3 x 3 and 4 x 4 squares on a calendar with the same tasks using a hundred board. What statements could you make about the patterns using different configurations of numbers? *Extension:* See Blackline Masters V - 2 and V - 3 for Calendar Math.

Notes and textbook references

K. Students can create unique sequences in many different ways. Examining the Fibonacci sequence is just one. Here is another. Each student chooses two different numbers, the first, A, between 100 and 200, the second, B, between 20 and 70. Using the Blackline Master V - 9 begin to complete the chart. Column A will decrease by one and column B will increase be one. Example: A = 167, B = 35.

A = 167	B = 35	sum	difference	product	quotient
167	35	202	132	5845	4.7714285. . .
166	36	202	130	5976	$4.61\overline{11}$
165	37	202	128	6105	$4.459\overline{459}$
164	38	202	126	6232	4.3157894. . .
163	39	202	124	6357	4.1794871. . .
162	40	202	122	6480	4.05

As students examine the sequences in each column, a variety of patterns and relationships reveal themselves. Why is the sum column constant? The difference column decreases by 2's. Why? What relationship(s) are evident in the product column? Is there a pattern among the quotients?
After looking at a variety of different sequences, students may wish to vary the numbers in columns A and B by a number other than one. As they explore these sequences, other "rules" for patterning can be established.

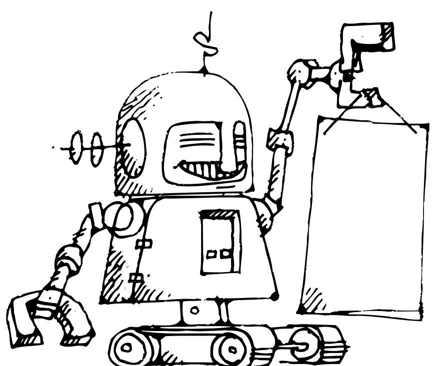
L. **Patterns in Pascal's Triangle** (Blackline Masters V - 6 and V - 7) These sheets point out patterns in Pascal's Triangle involving powers of two, the Fibonacci sequence, multiples, and modulus arithmetic. Students may want to visit a web site that shows Pascal's Triangle with the modulus coloring scheme. Here they can view up to 100 rows of the triangle colored according to multiples of 2 through 16.
(http://www.cecm.sfu.ca/cgi-bin/organics/pascalform)

Notes and textbook references

M. Have students work with "**Function Machines**." These machines take a number, set of numbers, or some other kind of *input* and do something (the same thing) to each *input* and "send out" an answer or *output*. The student's job is to figure out what the machine is doing to each *input*. For example, "**Function Machine I**" might take a number and multiply it by 3. Thus, if a student *inputs* 7, this machine would *output* 21. Students have the option to send many *inputs* through each machine in order to create a table and look for a pattern, and ultimately figure out the machine's function. "**Function Machines**" can get quite complicated. Such a machine might take a number and multiply it by three and then add 6. It would be possible to have machines that take polygons as *inputs* and *output* the number of diagonals in each polygon, while another machine might take singular nouns and *output* their plural forms. Students would probably enjoy creating physical representations of "**Function Machines**" and finding ways to operate them using their own patterns.

Note: A simple machine could be made by taking a milk carton or other box, cutting a slit in the top and bottom. Then "input ... output" cards are made by writing the input on one side of the card and the output on the other. When someone slips the card into the top slit with the input showing, it slides down, turns over, and falls out the bottom slit with the output showing.

In: 3, 5, 12



Out: 7, 11, 25

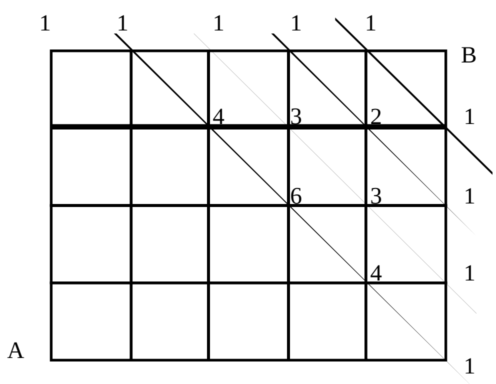
IN	OUT
apple	**2**
hop	**1**
strawberry	**3**
Mississippi	**4**
cannon	**?**
?	**4**

IN	OUT
square	**2**
pentagon	**5**
hexagon	**9**
septagon	**14**
octagon	**?**
?	**27**
?	**35**

Notes and textbook references

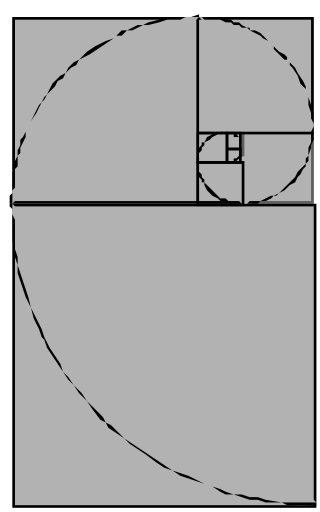
N. In counting by fives, the multiples end in 0 or 5. What happens when any number is multiplied by 5? by 25? by 55? Ask the students to make generalizations when you multiply by 10; by 100; by 1000; by 10,000; by 2; by 9; by 7; by 17; and by other numbers. In each case, describe the pattern you see.

O. **Problem Solving with Pascal** (Blackline Masters V - 4 and V - 5) Students solve problems all of which have solutions found in Pascal's Triangle. The stair problem, the ice cream problem, and the Twelve Days of Christmas problem generate numbers that may be found as a diagonal in the triangle. The pathways problem generates the numbers in the rows of Pascal's triangle as shown in the figure below.



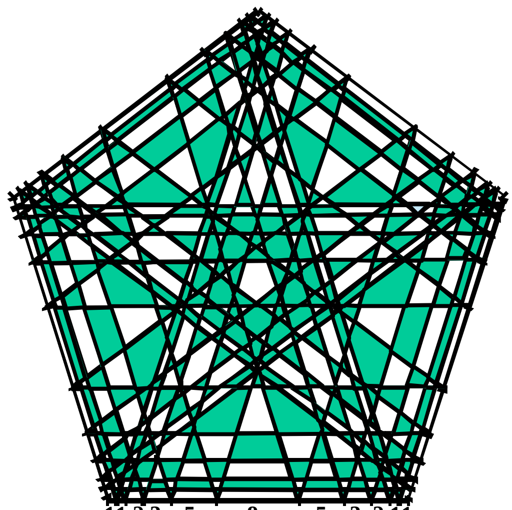
The numbers shown at each corner give the number of paths to point B if you may only travel up and right.

P. **Fibonacci Spiral** Students can generate squares with side lengths that match the Fibonacci sequence (i.e. 1,1,2,3,5,8, …). By arranging the squares so that each new square matches edges with the two next smallest squares it is possible to draw a quarter circle arc in each square so that the continuous curve drawn in all the squares makes a spiral.



Notes and textbook references

Q. **Fibonacci Art** Students can create striking designs through the use of the Fibonacci sequence. Start with any regular polygon. Mark off the sides of the polygon with segments whose lengths correspond to the Fibonacci sequence. The sequence will go up from 1, 1, 2, 3 to some suitable number and then retrace back down to 3, 2, 1, 1. (See the diagram below.) Line segments are drawn to connect corresponding points on the sides. Regions formed by these line segments should be colored in alternating colors.



R. There are some interesting patterns in the last (units) digits of the Fibonacci numbers.

Examine the <u>units</u> digit in each Fibonacci number -

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, . . .

Is there a pattern in the final digits?

0, 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, 7, 4, 1, 5 . . .

Yes! Have students turn their lined paper 90°. You now have some lined paper with long, thin columns.

Write the first 15 or so Fibonacci numbers at the top of each column.

Beneath that start another Fibonacci-like sequence 2, 2, 4, 6, 10, 16, . . .

Beneath that start another that begins 3, 3, 6, 9, 15, 24, 39, . . .

Keep going until you have completed all the sequences up to and including 9, 9, 18, 27, 45, 72, . . .

Now examine the units digit in each column. What patterns can you find?

Then examine the last two digits in each column. What are the patterns there? If you extend the sequences, you can then look at the last three digits and find the patterns there.

This can be a powerful entrance for students into the world of number patterns.

Notes and textbook references

S. Have students write out the first 15 or 20 elements of the Fibonacci sequence. Choose any three consecutive terms. Find the product of the first and third numbers. Now square the second number. What do you notice? Will this always happen? Try another Fibonacci-like sequence. What is the difference in this case? How is it related to the formation of the sequence?

1, 1, 2, 3, 5, 8, 13, 21, **34, 55, 89**, 144, . . .

34 55 89   34 x 89 = 3,026 55^2 = 3,025

4, 4, 8, 12, 20, 32, 52, **84, 136, 220**, . . .

84 136 220   84 x 220 = 18,480 136^2 = 18,496

21, 21, **42, 63, 105**, 168, 273, 441, 714, . . .

42 63 105   42 x 105 = 4,410 63^2 = 3,969

This exploration provides a wealth of opportunities for students to examine a variety of sequences.

T. Cut apart old calendars and give each pair of students a page. Challenge the pair to find as many patterns as possible. For example:

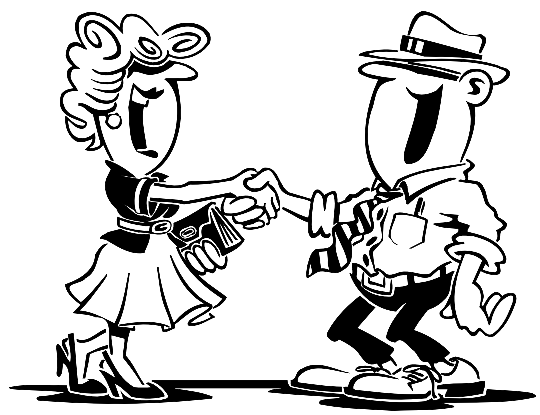
- sums of horizontal rows.
- sums of columns
- in a 3 x 3 array double the center number.

Then ask: ”Which pairs of numbers will have that sum?”

Notes and textbook references

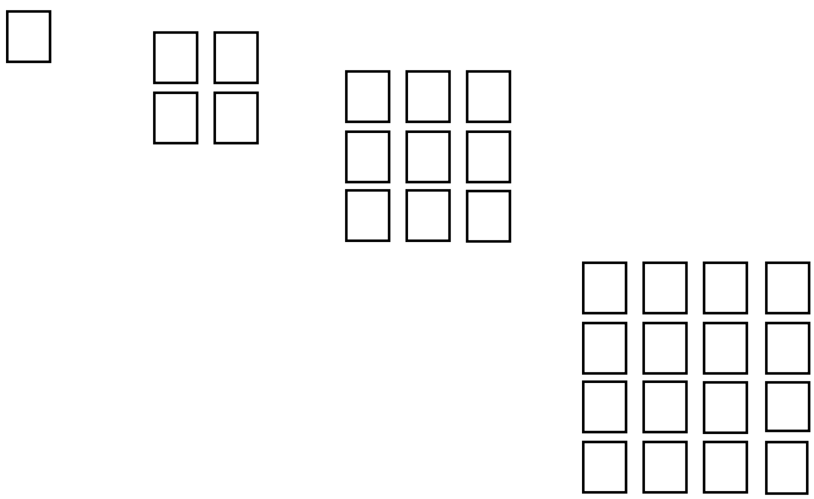
U. Have students fill in multiplication charts from 2 to 12 like the one in Blackline Master V - 10. Laminate the charts for prolonged use. Ask each student to pick a number from 2 to 12 and write it at the top of his or her chart. Then, using connecting cubes or other markers, cover all the multiples of the chosen number wherever the multiple appears. Have the students repeat the activity several times with other numbers and then compare the charts. Discuss why some numbers have more complex patterns than others.

V. Acting out problems like the handshake problem makes it easier for students to see the results. However, at the fifth grade level, it is a important to make charts to search for patterns and to make generalizations.



W. Link to Goal 4 by designing an investigation. What is the most popular selling television(or other appliance) in our area? Are the least expensive televisions the most popular? What features do consumers value most? Is there a pattern in the choices?

X. Give each group of 2 to 4 students 100 cubes or paper squares. Have them build "square" numbers by showing them square arrays of 1, 4, and 9 counters. Build models of other square numbers using the materials. Ask the student to extend the pattern by cutting the square numbers from a piece of centimeter grid paper. See Blackline Master III - 2 . Keep a record of the findings. Repeat the activity using cubes to model cube numbers. Build the numbers first using the cubes and then make models of the numbers by cutting the sides from the centimeter grid paper and taping them together. Keep a record of the findings. Discuss the patterns and how they relate to exponents.



Notes and textbook references

Notes and textbook references

5.02 Use algebraic expressions, patterns, and one-step equations and inequalities to solve problems.

A. Students have three types of school supplies in a box. The number of objects varies from student to student. "P" represents the pencils, "C" represents the paper clips, and "M" represents the markers. Give students different colored cubes or objects to represent pencils, paper clips, and markers as they solve problems with variables such as these:

Paulina's Box
8 pencils
5 fewer markers than pencils
22 things in all

P = _____ C = _____

M = _____

Jonna's Box
5 markers
3 times as many paperclips as pencils
21 things in all

P = _____ C = _____ M = _____

Reynold's Box

12 things in all
half are pencils
one fourth are markers

P = _____ C = _____ M = _____

Bill's Box
12 things in all
3 pencils
twice as many paper clips as pencils

P = ______ C = ______

M = ______

In	Out
1	4
2	6
3	8
4	10
5	12
n	2n + 2

B. On a coordinate grid, plot the following points as the class watches.

Day	1	2	3	4	5	6	7
ht. of plant (in cm)	7	14	21	28	35	??	??

You may want to have different students take turns plotting a point. The purpose is to show students how to take data from a table and use them as an ordered pair. Ask students to tell the rule that is used for this table. (height is seven times the number of the day or 7*n*.) Point out to students that the number of the day determines how the height. The independent variable is the day. In creating the grid, the day is the *x*-axis and the height is the y-axis. See Blackline Master V - 8.

C. In surveying the grocery stores, Jonathan found that apples varied in price. What would he pay for 6 pounds of apples at each of these stores:

Super Saver: 89¢ lb. **Corner Market**: $1.19 lb.

Big Discount: 79¢ lb. **Sam's Fruits**: $ 0.98 lb.

Show students that they could write these problems as an expression and fill in the different prices as they completed the problem above: "If *a* = the price for apples, then 6 x *a* would be the cost of the apples."

Extension: Have students repeat the activity, gathering costs from several stores and using a letter to represent the varying costs.

Notes and textbook references

*A **variable** is a symbol that can stand for numbers or objects. For example, "n" may represent the number of people who can ride in my car. The cost of supporting workshop participants could be figured as "n" people at $75 per person. Some variables represent a specific unknown such as x + 5 = 9. Other times they are used in formulas such as A = bh. Variables are also used to generalize patterns.*

Notes and textbook references

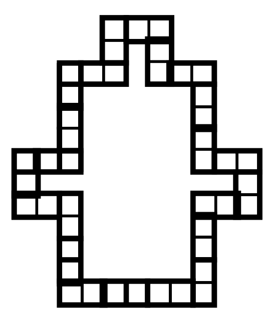
D. Have students explore situations in their environment that have natural variables. Have students gather and represent the data using questions like the following:

- How does the length of a student's foot vary with their height?
- How does school attendance vary with the day of the week?
- How does the number of hours of daylight vary with the time of the year?
- How does student age vary with grade level?
- How does student attendance at school vary with weather or month of the school year?
- How does the number of students in the school cafeteria vary with the time of day?
- How does the number of students buying lunch vary with the lunch menu?
- How does the speed of a swinging pendulum vary with its length?

Encourage students to come up with their own questions in order to add to this list. During this entire process, emphasize the search for any patterns that might emerge and encourage students to make "predictions" based on these patterns. For example, the average age of students graduating from high school would be 18 based on the pattern found when answering the question "How does student age vary with grade level?"

E. **Word Translation Square Puzzle** (Blackline Master V - 15) Students are provided with a scenario in which *T* represents the amount of money Tom has and *J* represents the amount of money that Joe has. Various word-problem situations are presented in which students set up expressions using the variables *T* and *J*. The small cards must fit together to make a larger square. Where two edges touch, a problem and its solution must match.

F. **Equation Dominoes**
(Blackline Masters V - 16 and V - 17) Students should work in groups to match each word problem to the equation which may be used to solve it. On each match, students should match the edges of the dominoes with the stars. Upon completion, the dominos will form a closed loop as shown here.



Notes and textbook references

G. Many "non-routine" problems offer opportunities to use "finding a pattern" as a problem-solving strategy. Whenever possible, model this strategy as one of the many approaches to solving problems. Some examples of the many rich problems include the following:

There is a village where all the streets are straight and perpendicular to each other. Each crossroad has one street light. How many lights would be needed if the village had 3 streets? How many would be needed for 20 streets? Could you generalize to any number of streets?

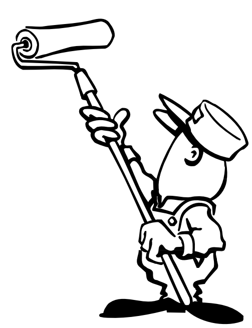
Lauren saw a sign along the interstate that said, "Gina's Pizza - over 100 varieties." If a plain pizza has tomato sauce and cheese, how many toppings does Gina's have available? Students can explore this problem by listing different toppings and creating each possible pizza. This listing process could get lengthy and complicated, thus the need for finding some kind of pattern.

This is similar to the classic problem of tracing all possible paths that a mouse might take to reach the cheese in a square maze of trails. See Blackline Master V - 13. The answer to this problem generates Pascal's Triangle (Blackline Master V - 14) if one lists the number of ways to get to each intersecting point of the maze. Note: the mouse travels <u>only</u> up and to the right; it does <u>not</u> meander back and forth.

Watch for rich problems and create a file!

H. Have students use hundred boards (Blackline Master V - 12) and counters. Cover all odd numbers with counters. Students go back and color <u>part</u> of the squares of the covered numbers red. Examine colored numbers for patterns. Look for other patterns by covering with counters on the board and then coloring to record:

1. Color even numbers yellow.
2. Color multiples of three green (note where they intersect).
3. Color multiples of four in blue (note intersection of evens and multiples of three.)
4. Color multiples of five purple.
5. Color multiples of six brown.
6. Color multiples of seven orange.
7. Color multiples of eight pink.
8. Color multiples of nine (student color choice).



Which numbers have the most colors?

Notes and textbook references

I. Perimeter and Area Patterns (Blackline Masters V - 18 and V - 19) Students examine some geometric patterns to discover number patterns relating to perimeter and area. They then use these patterns to find formulas to predict perimeter and area for larger figures.

The patterns created from these figures are each linear. It is fairly easy to use the repetitive pattern to find the answers required. Students may notice (or you may point this out) that patterns which increase by 2 every time n increases by 1 will have a formula of the form "$2n + ?$" If the formula value increases by 4 each time n increases by 1, the formula will be of the form "$4n + ?$" If you study the concept of slope, remind students of this example and explain how this ties in with slope.

Students should also be reminded of the geometric meaning of the variables. For instance in the first pattern, n represents the length of figure. Each figure has a top and bottom of equal length and ends of length 1. Pointing out this pattern, and then having students imagine what the 100th figure will look like, can help them find the formula. Show how this connects with the formula $2n + 2$ for perimeter.

Some students may be able to solve the problems by the iterative pattern only, others may know and use the rule about equal increment changes, others may see the pattern geometrically. Expose students to all three ways of viewing the problem and help them understand how they connect.

J. Block Patterns (Blackline Masters V - 20 and V - 21) Students examine geometric patterns to determine how they grow and to determine perimeter of the shapes formed. After completing a data table that shows pattern figure number and perimeter, the students should be able to find a formula to predict the perimeter of the n^{th} figure.

Notes and textbook references

5.03 Identify, describe, and analyze situations with constant or varying rates of change.

A. Have the students examine a variety of catalogs' order forms for mailing and handling charges based on amount of purchase. The relationships show a variety of changing rates - some constant and others varying. Students might also discuss the reasons for the differences.

B. Have the students record the temperature of a cup of water with an ice cube in it over the period of 20 minutes. Record the temperature every two minutes. You may want to use plastic, glass, and Styrofoam cups to test the differences in the rates the ice cube melts. Discuss the results and determine whether this was a constant or varying rate of change. Plot the data on a coordinate graph as an extension to see the how the rate of change is shown on the grid.

C. Design an experiment for determining if the student walks at a varying or constant rate of change. Your experiment will more than likely include stopwatches and marked measurements on the floor every few feet, yards, or meters. Students can determine if they walked at a constant rate by dividing the distance by time at each point along their walk to see if it is a constant number or plot points on the grid.