# The learner will recognize and use basic geometric properties of two- and three-dimensional figures.

*Notes and textbook references*

# 3

## *3.01 Use appropriate vocabulary to compare, describe, and classify two- and three-dimensional figures.*

**A.** Make a shape on the geoboard. Have your partner try to construct a figure congruent to it, looking at the figure. Count the sides, angles and vertices to see that the shape is the same.

**B.** Students should be encouraged to search through magazines for pictorial representations to include in a student-made book along with their own drawings and models from stir sticks. Make sure that these magazines are discarded ones or that students have received permission from parents before cutting.

**C.** Divide the class into groups and assign each group a polygon. Allow 15 minutes for each group to find as many objects as possible within the classroom or on the playground, etc., which are examples of that figure (i.e. rectangle-book, chalkboard, etc.). Have a spokesperson for each group stand up and state each example and why, using the correct vocabulary.

**D.** Have students create their own tangrams. Use this paper-folding activity (Blackline Master III - 8) to build vocabulary, make comparisons, and informally access students' understandings.

**E.** Use individual sets of tangrams. Have students take each piece (seven in all) and label the polygons. There are triangles, a square, and a parallelogram. They should identify the distinguishing characteristics of each piece. In small groups or with a partner, have students make and record the ways they use these polygons to create other shapes. See Tangram Blackline Masters III - 7 through III - 9.

*Notes and textbook references*

**F.** Using pattern blocks and the gameboard for **Apple-licious** (Blackline Master III - 10), have students play several times. After playing, have students discuss the strategies they used. Then incorporate the following ideas in their journals:

Describe your “”apple”” by identifying how many three-sided, four-sided, or six-sided figures you used.

Estimate (separately) the total number of angles . . . vertices . . . sides . . . that are in the design. Then, count to see how close your estimation was.

**G.** Discuss properties of polygons using pattern blocks and geoboards to create examples (number of sides and vertices, angles). Complete a chart such as this, with students helping to label columns.

| Shapes | Number of sides | All right angles | Sides equal |
|---|---|---|---|
| **PARALLELOGRAM** | 4 | SOMETIMES | OPPOSITES |
| **SQUARE** | 4 | YES | YES |
| **HEXAGONS** | 6 | NO | SOMETIMES |

**H.** Give students a geoboard and assign the problem of creating a shape that meets these three criteria:

1. Uses only one rubberband.
2. No crossovers with the rubberband
3. Must have area (space) within.

Before sorting the shapes by attributes, have the students verbalize what the similarities are of all the shapes created. All the shapes are polygons. Have all the students who have polygons with 3 sides go to the front of the room and display their shapes. Have the students tell the similarities and differences of the triangles created. Discuss number of sides, types of angles, the name of the shape, etc. Students come to the front of the room with their polygons based on the number of sides and continue discussing similarities and differences.

*Notes and textbook references*

**I.** Use geoboards for students to try these tasks:

- Make a 3-sided polygon with all sides different lengths.
- Make an 8-sided polygon that is not a stop sign.
- Make a 5-sided polygon that has at least two right angles.
- Make a polygon that has opposite sides parallel.
- Make a 3-sided polygon that has a right angle and two acute angles.
- Make a 6-sided polygon that has at least one obtuse angle.
- Make a 4-sided polygon that has no right angles.
- Make a polygon that has both vertical and horizontal symmetry.
- Make a polygon that looks the same no matter how the geoboard is turned - no fair using a square.
- Challenge - What is the maximum number of sides a polygon could have on a geoboard? Try sketching the polygon or using more than one rubberband to find out!

**J.** Students investigate how to use several triangles from the pattern blocks to make a large triangle, a four-sided figure, and a six-sided figure.

**K.** Using tangram sets, have students replicate the large triangle with various smaller shapes. Organize the search to determine how many different combinations are possible. Repeat with the middle-sized triangle. See Blackline Masters III - 7 through III - 9.

**L.** Play **Twenty Questions** with students trying to guess a mystery 3-D shape hidden in a bag or box. Questions should require a “yes” or “no” answer and should identify some property of a polyhedron (size or number of faces, vertices, edges).

**M.** After talking about three-dimensional models, ask students to choose one and to find as many examples as possible in their homes. Record to share with the class.

*Notes and textbook references*

**N.** What kind of Geo-pane can be made with a 3-D shape? Build a 3-D shape with toothpicks and 1 cm clay balls or mini-marshmallows. These may include a cube or rectangular prism. Hang each shape from a thread and dip completely into a water and soap mixture. Lift out and observe edges, faces, and vertices. (A recipe for a soap mixture is located in objective 3.01 T.) See Blackline Masters III - 11 through III - 13.

| | Edges | Faces | Vertices |
|---|---|---|---|
| Cube | | | |
| Square-based pyramid | | | |
| Tetrahedron | | | |

**O.** Play the **Who Am I?** game on the bulletin board. Display a list of statements such as “I have at least one square face”, “I have Five Vertices”, etc. In responding, students will notice that there may be more than one correct response and that each clue limits the range of responses. Direct the students to make up their own strips to be displayed on the board or exchanged with other students.

**P.** Using magazines, catalogs, and old books, have students bring in pictures of objects that are examples of polyhedra e.g., pictures of buildings, etc. Then, create a bulletin board by categorizing the pictures.

**Q.** Place objects that represent various polyhedra in a bag. Then, have individuals select one object from the bag. As each item is displayed, the class must write down the shape and list the object’s properties (number of edges, faces, vertices).

**R.** Provide small groups of students with a variety of polygons cut from card stock, paper the weight of index cards. These polygons could include squares, rectangles, and several different triangles. Some of the sides of these polygons should be equal in length, a side of one triangle might measure the same length as the side of the square, etc. Students could also cut out their own polygons to save teacher preparation time. Then students create as many different three-dimensional shapes as they can, by taping the various "faces" together. Groups write a description for each shape using as much vocabulary as possible, referring to the number of edges, faces, and vertices as well as specific names like pyramid, prism, etc. Students from other groups could try to identify a specific polyhedron based upon a written description. Groups could sort these shapes based on their own rules.

**S.** How many faces, edges, and vertices does a given polyhedron have? Use cubes, rectangular prisms, and pyramids.

**T.** **Soap Bubble Math**: Have students create polyhedra such as a tetrahedron from straws and pipe cleaners. To make a tetrahedron, cut six one-quarter sized sections of regular straws ( 2 1/2″ or 3″ lengths) and three pipe cleaners. Twist three pipe cleaners together at the top for about 1″. Slip two straw sections onto each pipe cleaner. Bend each pipe cleaner sharply in between every two straw sections. Attach one bottom end of each pipe cleaner to the middle of the next pipe cleaner (between two straw sections). Work all the way around.

To make a cube, cut twelve one-quarter-sized sections of regular straws (2 1/2″ or 3″) and four pipe cleaners. Make two separate squares using four straw sections and one pipe cleaner for each square. Then cut your remaining two pipe cleaners in half. Use one piece of pipe cleaner and one section of straw to join the two squares at each corner.

Dip these polyhedra into the following soap mixture and observe the edges, faces and vertices created by the soap film. Have the students observe and record what happens when the soap film meets in the interior of the polyhedron.

***Soap Bubble Solution:***

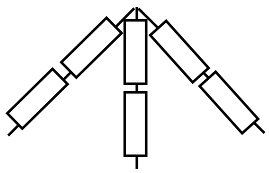
1 cup dishwashing liquid soap (Joy™ works best)
1 gallon water
1 tablespoon of glycerine (makes stronger bubbles)

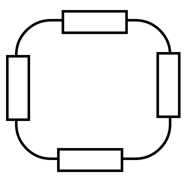
Add soap and glycerine to cold water and mix gently trying to prevent suds from forming. Age for five days for best results.

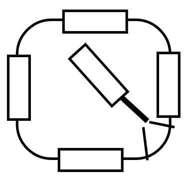
*Notes and textbook references*

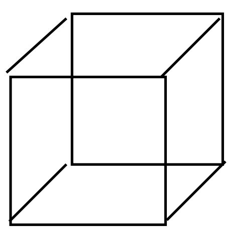
***Polydrons™*** *are plastic shapes that snap together and create a variety of three-dimensional shapes. These polyhedrons come in a variety of shapes including squares, parallelograms, triangles, hexagons, and pentagons to mention a few. Most students find these materials very engaging.*

*Notes and textbook references*









**U.** Use toothpicks and gumdrops or marshmallows to make models of two- and three-dimensional figures.

**V**. Given models of a cylinder, a square-based pyramid and a rectangular prism, describe the ways they are alike and the ways they are different.

**W.** Tell how a cube and rectangular prism (not a cube) are alike and different.

**X.** One student thinks of a shape. The others ask (yes or no) questions about its properties, trying to guess it. Does it have right angles? Does it have three sides?

# 3.02 Use a rectangular coordinate system to solve problems.

# a.) Graph and identify points with whole number and/or letter coordinates.

*Notes and textbook references*

**A.** **Where's your neighbor?** Create a poster-sized (or a shower curtain and colored electrical tape) rectangular coordinate grid. Label the horizontal axis (letters or numbers) and vertical axis (numbers only). Have students write their names on index cards and tape them to points (one card per point). Next call out students names and have the students identify the coordinate points of that student's name.

**B.** **Musical Points** Musical Points is an adaptation of musical chairs. Either use the shower curtain grid from activity 3.02a) A or create a rectangular grid (with the horizontal and vertical axes labeled) that can be used on the floor. Write the name of all of the coordinate points that are labeled on the shower curtain (or floor grid) on individual slips of paper. Put them in bowl. Play some music. As the music plays have students walk without stopping around the different points on the grid. Without warning the students, stop the music. The students must "freeze" on the point that they are standing when the music stops. Pull one of the slips of paper out of the bowl. The student standing at that point is eliminated and must sit on the edge of the grid. Continue until only one child is remaining on the grid.

**C.** Have students create their own coordinate puzzles like the "Expressions of Appalachian Mountain Folk". (See Blackline Master II - 7.) Students decide upon a list of words to include in their puzzle. This list of words might revolve around a specific theme, for example, words worth $100, North Carolina state symbols, math vocabulary words, etc. All the letters contained in the word list are written at points on a grid. The students can decide how large to make their grid. Then, students write a clue and response sheet. Each word is defined, and the letters of the words are identified by coordinate pairs. Suppose that a group of students have decided to create a coordinate puzzle using vocabulary related to this strand. One of the terms in their list might include "range." The clue for this term might look like this:

________________________________________

(6,7) (3,4) (3,6) (2,2) (1,1)

*The difference between the largest and smallest numbers in a set of data.*

Of course students will trade puzzles and solve them. Vocabulary lists from any content area could be reviewed in this manner.

*Notes and textbook references*

**D.** Have students collect temperature or precipitation data for one month. Write ordered pairs which include the day of the month and the temperature or precipitation amount. Graph the ordered pairs on a coordinate plane. Connect the dots to create a line graph for these data. Extension: Students should draw conclusions based upon the shape of these data and generate predictions about the next week. After several weeks, compare the two months of data for trends, similarities and contrasts.

**E.** Give students Blackline Master II - 7 "Expressions of the Appalachian Mountain Folk" - a puzzle which they decode using ordered pairs.

**C.** Given the following coordinates, plot the points in order on cm graph paper (See Blackline Master II - 5). Encourage students to create other designs.

(13,3) (15,5) (17,7) (19,9) (20,11) (21,13) (21,15) (20,17) (19,18) (17,18) (15,17) (13,15) (11, 17) (9,18) (7,18) (6,17) (5,15) (5,13) (6,11) (7,9) (9,7) (11,5) (13,3)

As a challenge, give students Blackline Master II - 6, "Using Coordinates", which includes fractions in the ordered pairs and looks at doubling the numbers, etc.

**F.** Since maps are labeled with these same kinds of ordered pairs, map reading is a natural part of developing this objective. Also, the Sunburst™ software program "Safari Search" provides great practice in locating positions on a grid while solving increasingly more difficult logical thinking problems.

Observe students playing the game described here. Watch for their ability to locate and name grid positions. Watch for students who are thinking and applying strategies as opposed to those who are randomly asking for clues. Notice the speed with which students are able to locate positions. Although students will not be expected necessarily to perform the task under timed conditions, speed can sometimes indicate the level of skill development.

Notes and textbook references

**G.** Have students look at different maps, noting whether the labels mark the grid lines or the blocks created by the lines. Why might labeling the spaces rather than the lines be helpful? Which way are North Carolina road maps marked by the Department of Transportation? Use a map from the social studies book or draw one. Label the horizontal axis with letters and the vertical axis with numbers. Ask children to tell you what is located at certain positions on the grid.

**H.** **Map Making.** Have students create a treasure map using a grid and labeling the squares with letters and numbers along each axis. Have children write at least 4 clues on an index card which will help their friends locate the coordinates of the treasure. The answers may be written on the back of the card for self-checking. Put a directional (compass) indicator on the map. Make the map interesting by putting land marks on it but do not show the location of the treasure. See Blackline Masters II - 4 and II - 5 for grids.

**I.** Know your community. Divide students into groups. Give each group a local map. Working as a team have students use map grids to locate points of interest and record. Use this information to create an index for the map.

**J.** Put masking tape on two sides of a geoboard to label rows and columns. Students work in pairs. Each student has a geoboard. Students take turns hiding a “fruit loop” or other ring on a peg. Students take turns guessing coordinate points to find opponent’s fruit loop. (This activity is similar to Battleship).

**K.** Students work in pairs. Each student has a copy of the same coordinate grid. One student places a small object at a point on the grid. Separate students with a file folder so that they cannot see each other’s pages. The second student tries to guess where the object is hidden by naming ordered pairs. First student responds to each guess by telling opponent in which direction to move with the next guess. Students keep track of how many guesses it takes to find the hidden object. During different seasons it is fun for students to “hide” seasonal objects.

*Notes and textbook references*

**L.** Use commercial games and software such as "Battleship," "Bumble Plot" and "Gertrude's puzzles."

**M.** Give each student a copy of a coordinate grid with the letters of the alphabet at various intersections. (See Blackline Master II - 7 Secret Code Grid.) Write a secret message for them to decode using the ordered pairs. For example:

(5, 6) (3, 7) (1, 10) (7, 2) (6, 4) (9, 8) (3, 7)      (3, 10) (6, 4)

(9, 8) (4, 5) (3, 10) (1, 5)      (7, 2) (1, 10) (4, 5) (10, 3) (10, 3)

**N.** Have students use graph paper to make their own secret word coordinate graph. Have them label points on the graph with alphabet letters and write ordered pairs for each letter of their secret word. (See Blackline Master II - 7)

**O.** Have students place buttons, beans, Cheerios™, or Life Savers™, etc. on a grid hidden behind a "shield" such as a manila folder. Then a partner tries to locate the object by naming ordered pairs on the grid. The clue given indicates which direction to move to get closer to the object. These clues might use compass directions such as north, south, east, and west or others like up, down, right, and left. Ask students to come up with their own ideas about how to vary this game.

Observe students as they play the various versions of this game. Watch for possible confusions about which coordinate in the ordered pair comes first. Some students find it easier to remember to go over or right first if they think of an airplane taking off.

*Notes and textbook references*

**P.** There are many engaging games that provide students opportunities to practice this skill. Here is an adaptation of the game **Coordinate Dice** from Math For Girls and Other Problem Solvers, an EQUALS publication. Each pair of students will label a 12 by 12 grid with numbers on both coordinates. They will also need two pairs of dice. These dice need to be in two colors, for example, 2 red and 2 green. The coordinates of the grid are then labeled to match the colors on the dice; for example, the horizontal coordinate could be labeled red and the vertical coordinate green. Each student also needs some markers which are different from his/her partner's markers. To play this game, students take turns rolling the dice. If a student rolls a 2 and a 3 on the red dice and then a 1 and 5 on the green dice, this determines the point (5,6) on the grid. The student rolling (5,6) then places one of his/her markers on that point. Students proceed in this manner, taking turns until one of the players has 5 markers lined up in a row, horizontal, vertical, or diagonal. If a player rolls an ordered pair that is already covered, then he or she loses that turn. Obviously, there are points on the grid that will not be possible to cover because of using a pair of dice. These points include all of those with 0 and 1 as part of the ordered pair, such as (1,0) and (5,1). One of the rules might be that if a player rolls a coordinate point that is already covered, he or she may choose to place a marker on one of these "impossible-to-cover" points. Have students play this game before pointing this out, and then ask them whether they notice points being left out. Then suggest, or perhaps a student will suggest, this new role. Have students create new versions of this game. Is there a way to change the rules so that more strategy and less luck is involved? When a game is completed, have students record the ordered pairs of the 5 points creating the line, for example, (4,4), (5,3), (6,2), (7,1), (8,0). This will help make the connection between the actual position on the grid and its paper and pencil representation. See Blackline Masters II - 4 or II - 5 for grids.

**Q.** Can you draw this picture? Directions: start at (2,3) and draw lines connecting these points in order .... (2,4) (1,5) (2,6) (2,7) (4,7) (4,6) (6,6) (6,5) (8,7) (8,3) (6,5) (6,3) and (2,3). Then draw a line from (3,3) to (3,2) and another from (4,3) to (4,2); then add a small dot in the center of the cell surrounded by (2,5) (3,5) (3,6) (2,6). Does your picture chirp? See Blackline Masters II -8 and II - 9.

*Notes and textbook references*

**R.** Give students a 10 by 10 grid (Blackline Master II - 10). Students will focus on the spaces rather than the points of intersection. "Hide" three rectangles on your grid by coloring a rectangular area red, another green, and a third yellow (you can use any three colors). Remember that a square is a special kind of rectangle. Report the area of each rectangle in square units to the class; for example, the red rectangle is 20 square units, the green is 25 and the yellow is 30. The students try to determine where each rectangle is on the grid by asking for clues. As a student names a cell, for example, (E,1), the color of that cell is reported. The goal is to locate each rectangle with as few clues as possible. Stop after a few clues have been given and ask students what they know for sure. Discuss what possible shapes each rectangle might have; a rectangle that is 25 square units could be 1 by 25 or 5 by 5. Since the grid is 10 by 10, the only possible shape for this rectangle is 5 by 5. This activity provides opportunities for spatial visualizing and recognizing factors. After modeling this game with the entire class, have students play with a partner or in small groups.

**S.** Play a variation of the rectangle game in 3.02a) - R by coloring one rectangle on a 10 by 10 grid. This time the grid is labeled on the coordinate lines entirely with numbers. The rectangle is placed anywhere on the grid and the area of the rectangle is reported. Students try to find this rectangle by gathering clues. As students name a grid location such as (3,4), they are told whether that point is on or outside of the rectangle. Again, the goal is to locate the rectangle with as few clues as possible. It may be helpful to add a third kind of clue indicating when a point is on the perimeter of the rectangle. This is a variation of the game "**Where's the Rectangle**", also from <u>Math for Girls and Other Problem Solvers</u>. While modeling the game, with the entire class, be sure to ask questions like "What would be really helpful to know at this point?" Be sure students have access to colored square tiles or connecting cubes to move around on grids. As students play this game in pairs or small groups, emphasize searching for strategies. See Blackline Master I - 19.

*Notes and textbook references*

**T.** When first playing the game in 3.02a) -S, students may find using colored square tiles or connecting cubes helpful as they move and adjust rectangles on the grid. Extensions of this game could include using 4 or more rectangles and larger grids. Ask students whether there is a strategy for locating the rectangles with a minimum number of clues. How does this strategy vary with the number of clues. How does this strategy vary with the number of rectangles hidden on the grid? This is a variation of "Multi-Color" from *Math for Girls and Other Problem Solvers*, an EQUALS publication.

**U.** Students can reassemble pictures that have been cut into square units and labeled with ordered pairs. For example, a large picture from a magazine could be cut into one-inch squares with any "extra" dimensions becoming a frame. Each square would be labeled on the back with an ordered pair that indicates where that square fits into the grid. The bottom left corner square would be (A,l), and the square to its right would be (B,1) and so on. If ordered correctly, all the squares would fit together to form the original picture. Students could create their own pictures and cut and label them the same way. Or, a small drawing could be enlarged through the following process: Cut a small, simple drawing into square inches. Give students one of these squares and a larger blank square measuring four inches. Each student writes the ordered pair of the smaller square on the back of the larger and then draws on the front of the larger square whatever appears on the smaller. All of these larger squares are then reassembled in proper order to create an enlargement of the smaller picture. Discuss the degree of magnification of each picture, a nice connection to multiplication. This is the same strategy used to enlarge or shrink pictures and maps.

**V.** Have students play **Place the Point**, a simple graphing game that uses number cubes and a first quadrant grid. See Blackline Master III - 14.

*Notes and textbook references*

## *b.) Describe the path between given points on the plane.*

### A. Not So Fast

Display transparency of Blackline Master III - 1

Version 1: Students work together to gather up all the balls and bats after the game. Two number cubes are rolled. On the first roll the students start at (0,0). They discuss which number on the dice to use as the first or horizontal coordinate, and which to use as the second, or vertical. The object is to get the balls and bats picked up with as few turns as possible. In order to pick up (or mark out) a ball or bat, students must land exactly on the point where the item is located. With the next roll of the dice, *students must start at the point where they landed on the last move*. Again, they discuss which number to use first, and whether to move left or right; should they move the second number of spaces up or down? Play continues until the whole field has been cleared.

Version 2: Students play competitively, with one collecting balls, and the other collecting bats. If a student moves incorrectly, his opponent says, "Not so fast," and points out the error. (eg. Moving vertically first) The player loses a turn.

Version 3: Using version 1 or 2, players are allowed to collect any objects they pass on the move, thus allowing several items to be collected on one turn.

It is important for students to record moves, at least in the beginning. Once they are good at the games, it becomes boring and you may decide to skip that step.

### B. **Four in a Row** is a game for two students or teams.

Materials: First quadrant grid, Blackline Masters III - 2 through III - 5
Set of Number Cards, cut apart and in two decks

Object of the Game: Claim four points of intersection in a row, in any direction

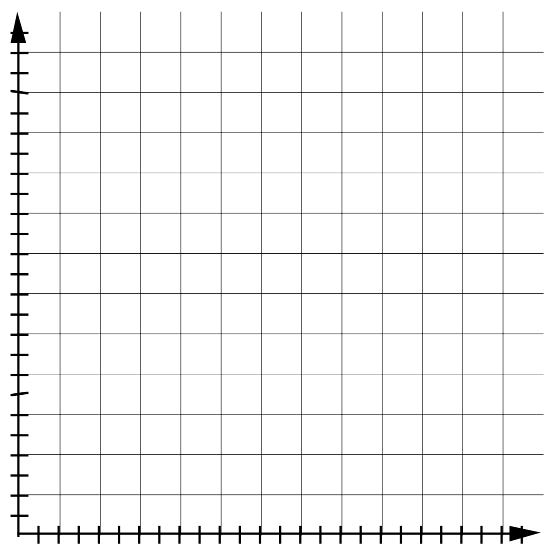
Play begins with player one drawing a card from each stack, recording them as an ordered pair with the East/West card the first point and the North/South card second. Beginning at (0,0) the player locates her new point and claims that intersection with her initial. The second player now takes a turn, moving in the same way.

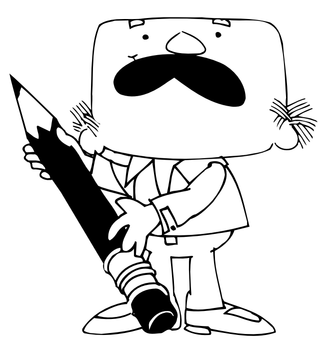
On the next move, player one again draws two cards, records the ordered pair, and claims a new point. *However, on this move the player begins on the point she claimed in the last move*. Player two does the same.

On each following move, the players, in turn, draw and record ordered pairs and move, beginning *at any point that player has already claimed*. A player must consider where to start and whether to move East or West and North or South. Should she try for a point or try to block her opponent?

The first player to claim 4 in a row wins. A player may not claim a point already claimed by another player.

**C.** Draw a coordinate grid and label the coordinates. Draw a dot-to-dot picture on this grid. Then write directions using ordered pairs so that another student could draw your picture. See Blackline Masters II - 8 and II - 9.





*Notes and textbook references*

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**D.** **Treasure Hunt** Provide the students with Blackline Masters II - 8 and II - 9. Have students label the bottom left corner of the grid with a zero and then label the *x*- and *y*-axis 1-10. Next have the students create a treasure map by hiding five gold coins, three silver coins, and two diamonds in the grid. To play the game: A team of two students begin playing by one student guessing the location of his teammate's treasure by calling out an ordered pair. The student must describe the path between the given points by using directional words. For example, the one student may tell the other student to go East five units and North six units (5,6). If the location is a hit, the student earns the treasure. If not, his teammate calls "miss". Each student is allowed a total of 20 guesses, with points earned for each treasure discovered: ten points for a gold coin, five for a silver coin, and 25 points for a diamond.

**E.** **What's your point?** Pairs of students play this game with one coordinate grid and three spinners. Each player spins the polygon spinner to determine which shape they have to complete. Then, in turn, each student spins the coordinate spinners to determine which point he/she will graph. Players may use any point that has been graphed and the the first to make his/her polygon is the winner! See Blackline Masters II - 8, II - 9 and III - 17