The learner will understand and use data and simple probability concepts.

4

4.01 Gather and organize data from surveys and classroom experiments, including data collected over a period of time.

Notes and textbook references

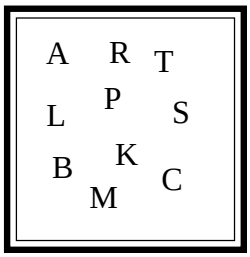
A. Data investigations are logical ways to integrate with other areas of the curriculum by having students generate or gather data to answer a number of questions:
• Science - weather records, pulse rates, plant growth, results of experiments, surveys, environmental studies.
• Social Studies - where students were born, states traveled in by students, types of businesses in your community, population of grade levels in school, information from newspapers.
• Language Arts - Favorite books and fairy tales, books read.
• Personal information - shoe sizes, birthdays, favorites.
• Physical Education - amount of time to walk a given distance, number of jumping jacks completed in a given time.

Activities in this section focus on data collections and organization. These are the first steps in a data investigation.

Data collection can be done through observations, surveys, questionnaires, interviews or polls.

B. Have the students make a "live" circle graph. Collect the data. You could use a tally chart to organize the data. Prepare the center point of the circle by using a plastic container such as an ice cream tub. Cut a slit in the lid of the tub. Put a ball of colored yarn in the tub for each item you will be graphing. Pull the yarn through the slit in the tub. Organize the students by where they fit in the data. For example, if you were graphing favorite cookies, have all of the students choosing chocolate chip cookies to line up. Give the first person in line the end of a piece of yarn. Then line up the students choosing oatmeal cookies. Give the first student in line the end of a piece of yarn. This group will follow behind the chocolate chip group. Continue until you have all of your students in a circle. The students holding the yarn may set the yarn on the floor. Students can then see the circle graph come to life. You may want to have labels written to put into the circle to show which part of the graph represents which type of cookie.

Notes and textbook references



C. Have children write their first initial on a large piece of paper. As a class have children decide how to organize this information so that they can determine the most common first initials in the class. Variation: Choose other topics such as last initial, birth month, favorite day of the week, birthday.

D. Gathering information necessitates asking good questions. Divide class into two groups, with one group gathering data from question one and the second group gathering data from question two. Have children discuss how easy or hard it is to organize the information, the completeness of the information, and the purpose of the surveys. Because this is an open-ended activity, encourage all students to express their opinions.

1. What is your favorite food to eat for dinner?
2. Would you prefer a hamburger, a hot dog, spaghetti, or fried chicken for dinner? OR

1. How much allowance do you think adults believe a third grader should be given each week?
2. Which of these allowances do you think adults would say third graders should be given each week: $1.00, $3.00, $5.00, $10.00?

Extension: Have students brainstorm information they would like to gather. Discuss how to write questions that will give them usable data and be easy to record (graph). Be sure to discuss whatever guidelines they suggest. Ask for explanations!

E. Is our classroom well-supplied with a variety of books? Have students talk about ways to gather and organize this information. For example, one group might establish categories such as fiction, biographies, weather, countries, and so on. Sort books into these groups and then count books. *Allow different groups of students to make and carry out a variety of data gathering plans to answer the question above.*

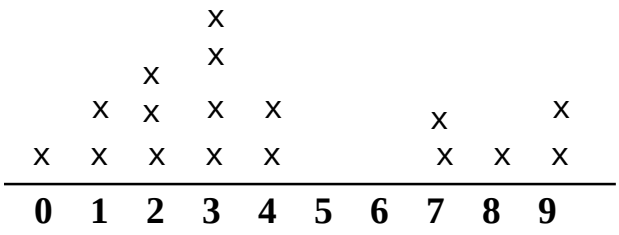
F. Have several students work with the attendance person to gather information about the number of absences by grade levels and by days for four weeks. Divide the class into groups, giving the same data to all. Tell students that you want to consider two things: absences by grade levels and absences by days of the week. Have them sort the data and decide how to display it. Let each group present their display to the class.

Notes and textbook references

G. Have each student complete the Student Census (see Blackline Master IV - 1). Put these into one notebook for students to use in subsequent activities. For example, divide the class into groups. One at a time, each group decides upon a question it wants to investigate using the census notebook. (How many students have 6 or more letters in their last names? What zip codes do we have and how many students have each?) Have students report to the class their method of keeping track of the data.

H. Use the following as sources for charts and graphs you provide for your students: *USA Today*, Guinness Book of World Records, ingredient labels on cereal boxes, soup cans, and so on. Give students a chart or graph. Ask them to list the things they know based on the data. What might they conjecture?

I. Ask students whether they think all digits are used with equal frequency in their telephone numbers. Have them explain their answers. Should we consider the first 3 digits? Why or why not? Divide class into groups and have each group graph the frequency of use of the last four digits of their phone numbers. Each group may create either a bar graph or a line plot. A line plot looks like this:



After the groups share their graphs, have each group decide how they could combine the data from all of the groups (add with pencil and paper, use calculator, make a giant graph). After the class has discussed the process, have one group of students (volunteers or appointed) collate the data and create the display. *Follow up:* When the display is finished, ask all students to write about the activity and make summary statements about what they discovered. If you lived in another part of North Carolina, do you think the results would be the same? *Extension:* If you each chose telephone numbers of local businesses, would the results be the same? Have students discuss their thinking, each select a business number at random, and graph the results. Why might businesses prefer numbers ending in zeros?

Notes and textbook references

J. Cut out daily weather maps from *USA Today* or other newspapers. Have students discuss temperatures and temperature changes. Save maps and make monthly comparisons. Compare daily temperatures in different cities and discuss how you would dress in different cities.

K. The owners of a concession stand are planning to buy soft drinks for a booth near your school. They are on a very tight budget. They want to stock only the drinks that sell well, but they do want a variety. You and your partner are the sales representatives who will help them decide what to buy. Gather information, put it into an appropriate graphic display and tell (write) what you think the order should be.

L. At this point in time, students have created many representations of data. Having students write their own test questions provides much insight into their understanding. Working in pairs or small groups, have students select several charts or graphs and write questions about them. They will also answer their own questions. Then, these questions are used as an assessment tool. Some of these questions will be surprisingly difficult as students work to challenge their peers. Each "test" question could be answered on a separate sheet of paper so that the group that wrote the question can correct all the responses and then explain their question and answer to the class. Encourage students to find ways to reinforce all the positive aspects of any student's answer.

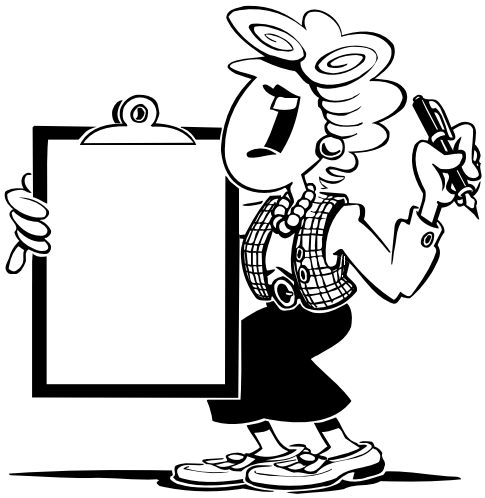
M. Create a "student census." Begin with a discussion of "What is a census and how is a census conducted?" "How is the information used?" Lead a brainstorming session to generate a list of questions that might be included in this student census. This could be done with the entire class or in groups of three or four followed by a general sharing with the entire class. Students should be encouraged to list all the things they would like to know about themselves and their classmates. This might include some of the following: first, middle, and last name; birthdate; age; birth place; address; phone; number of people living at reported address; number of brothers; number of sisters; hair color; eye color; handedness; hobbies; grades attended at this school; special talents; favorite school subject; favorite pastime; number of hours spent watching TV during the week; number of hours spent on homework; etc. When all ideas have been recorded, have students prioritize and order the list. You need to remind everyone at this point about questions which might be perceived as an invasion of privacy. Students should be encouraged to

eliminate from the list any item that someone might feel uncomfortable answering.

Create, or have a few students create, a census form that everyone in the class will complete. All of these census forms will be filed alphabetically in a class census folder. As any information changes, students are responsible for updating their census form.

Small groups or pairs of students then choose a specific area to organize in some way. This process may lead to further questions or comparisons. For example, do boys or girls watch more TV in our class? As students try to organize information, discuss the ease with which certain types of data can be organized. Those students organizing things like "favorite pastime," might report having more difficulty than those who are organizing "handedness." What might be done to simplify gathering data about things like "favorite pastime"? Do students want to create a new survey with a different question?

Some students may be interested in comparing their class to others in the school or even another school. This would require surveying other classes and perhaps writing to another class in the area or another part of the country. This would be an ideal time to use electronic mail to send data.



Notes and textbook references

Notes and textbook references

4.02 Determine the number of permutations and combinations of up to three items.

3, 4, 5

permutations:	combinations:
3, 4, 5	3, 4, 5
3, 5, 4	
4, 3, 5	
4, 5, 3	
5, 3, 4	
5, 4, 3	

Permutations are arrangements or lists where the order is significant or important. Ex. The digits 3, 4, 5 can be arranged in six different ways to form telephone exchanges or house numbers. The order is important. There are *six* **permutations** possible using these three digits.

Combinations are arrangements or lists where the order is neither important nor significant. Ex. If I am adding 3, 4, and 5 the sum is 12. The order in which I write the addends is not important. There is *one* **combination** of these three digits to make a sum.

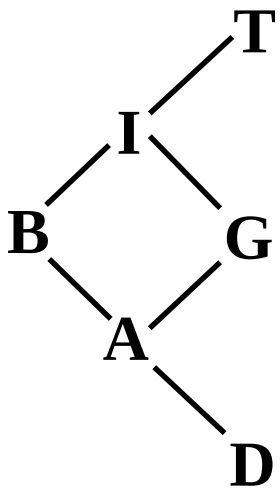
A. Have students take twelve counters and show all possible ways to arrange them on three squares of white paper. Suppose you arranged the twelve counters on three squares. (For this activity 2 + 3 + 5 is a similar grouping to 5 + 3 + 2 and is not counted again).

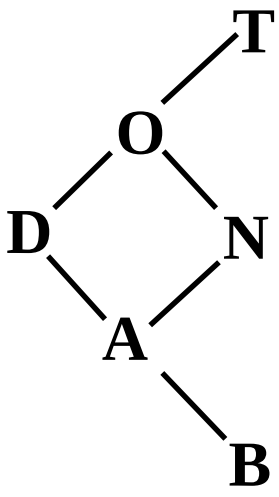
B. Divide the class into small groups. Give each student an index card to write his/her name on. Tell each group that a photographer wants to take pictures of them in groups of two. Have the group figure out how many pictures the photographer will need to take and who will be in each picture. Students can use the index cards as manipulative. Remind students that a picture of Bobby and Sue is the same as a picture of Sue and Bobby. When students have completed this have them find out the same information if the photographer wanted them in groups of three.

Notes and textbook references

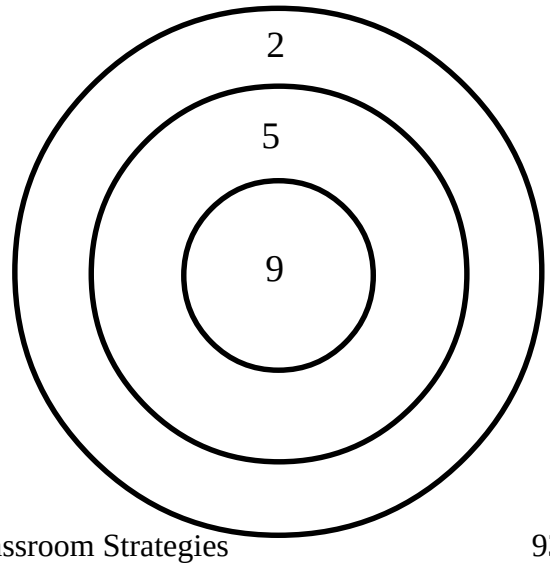
C. Make a set of cards using an index card for each group. Write on the cards the following items: PB&J sandwich, ham sandwich, apples, oranges, milk, and juice. Divide the class into partners or small groups. Each group needs to figure out the kinds of lunches that can be packed into a lunch box. Each lunch needs to have a sandwich, fruit and a drink. Have one student record for their group.

D. How many different three-letter words can you form when you read the triangles (tree diagrams) from left to right? Help students to write the words from these examples and then have students create other tree diagrams.





E. Draw a large bull's eye as shown. Tell students that three darts hit this target. How many different possible scores could the player have earned? (It does not matter in what order the darts landed). Make an organized list of the possible totals. See Blackline Master IV - 2.



Notes and textbook references

F. Group the students in threes. Have each person write their name on an index card. Tell the students they are at an amusement park getting ready to ride the roller coaster. Each seat holds one person. Each person wants to sit in each seat. How many times will they have to ride so that each person gets to sit in each seat?

G. Dipping ice cream cones. Cut out circles (scoops) in three different colors and triangles (cones) in one color. Challenge students to create as many different double dip cones as possible. Discuss whether or not order is important. Some students will say no, others yes. "I like my vanilla on top to drip into my chocolate!"

4.03 Solve probability problems using permutations and combinations.

Notes and textbook references

A. Have the students find the probability of getting two heads, P(h,h), when tossing two pennies. Students will need to find all possible outcomes in order to determine the probability.

Example: head, tail
tail, head
tail, tail
head, head

the probability of head, head is 1 out of 4

$P(h,h) = \frac{1}{4}$

B. Play the game **Chips in a Cup**. Give each student a paper cup and three two-colored counters. Have the students shake the cup then pour the chips out 6 times. Have the students record the results. Students can then predict the probability of getting three chips of the same color using the vocabulary: likely, unlikely, impossible, and certain.

Extension: Have students use a tree diagram to determine the possible combinations of three two –colored counters. Students can then compare their prediction with the results of the tree diagram.

C. Choose three colors of cubes. Put one of each color into a bag. Prepare three bags. Have the students choose a cube from each bag. Record the results. After a few results have been recorded pose the problem: What is the probability of getting the same color cube?

D. Have students use notebook paper to make a grid with three columns. Students will then choose two markers/colored pencils of different colors. Using only those two markers have the students discover how many ways to color a row of three boxes. Once this step is completed have the students cut apart the row and place them in a bag. Draw a row out of the bag and record. Continue a few times. Have students find the probability of pulling a row of two of the same color.

Notes and textbook references

E. Make two spinners divided into thirds. Label one spinner red, yellow, and green. Label the second spinner with the numbers 2, 4, 6. Have the students find the probability of spinning each spinner one time and getting "red 4." See Blackline Masters III - 15 and III - 16.

Extension: Use three spinners divided in halves. Label with letters, color and number. Find the probability of spinning "green B 6."

To make a spinner: use a four-inch piece of cardboard cut into a square. Cut a circular spinner top. Divide the spinner top into thirds. Label each section appropriately. Find the midpoint on the back of the cardboard square. Poke an unfolded paperclip through the midpoint of the square. Tape down the paperclip to the back of the square. Put a button on the paperclip to act like a washer. Then poke the paperclip through the center point of the spinner circle. Fold the paperclip down and put a small piece of tape on the end. Draw an arrow on the square card in one corner to be the pointer.

F. Put number tiles 2, 5, 3 in a bag. Have the students take out three numbers one at a time. Place the numbers in order (left to right) on the table as drawn. What is the probability that the number will be less than 350? What is the probability the number will be even?

G. Put the letters a, t, r in a bag. Take out three letters one at a time. Place the letters on the table in the order drawn. What is the probability that an English word will be made?

Some thoughts on graphing . . .

Notes and textbook references

We live in an information age. Rapidly expanding bodies of knowledge combined with increased uses of technology clearly indicate that we as adults and the children currently enrolled in elementary grades will need to be able to evaluate and use vast amounts of data in personal and job-related decisions.

Skills in gathering, organizing, displaying, and interpreting data are important for students within all content areas. Graphing activities incorporate knowledge and skills from a variety of mathematical topics and integrate geometric ideas with computational skills, and classification tasks with numeration understandings.

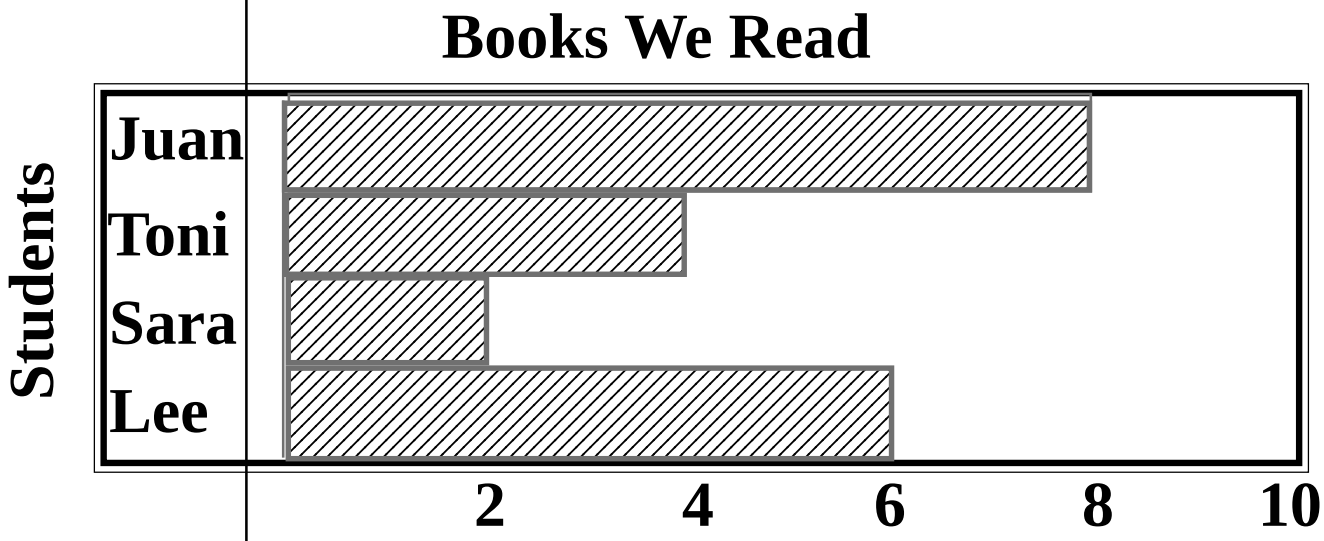
Graphs provide a means of communicating and classifying data. They allow for the comparison of data and display mathematical relationships that often cannot easily be recognized in numerical form. The traditional forms of graphs are picture graphs, bar graphs, line graphs, and circle graphs. New plotting techniques include line plots, stem-and-leaf plots, and box plots.

The following discussion is paraphrased and condensed by permission from the National Council of Teachers of Mathematics from *Developing Graph Comprehension*, pages 1 - 9. Information on newer techniques is for the teacher's benefit, since most applications of these techniques are more appropriate for upper elementary and middle grades rather than third grade.

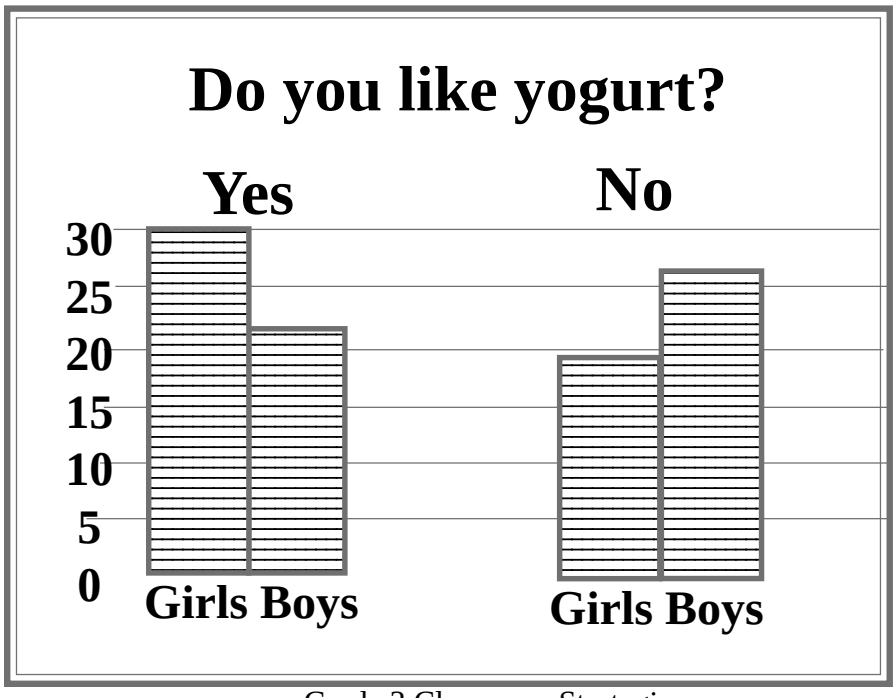
Notes and textbook references

Traditional Graph Forms

Bar graphs. Used horizontally or vertically, bar graphs (also called bar charts) compare discrete quantities expressed by rectangular bars of uniform width. The heights (or lengths) are proportional to the quantities they represent. The bars are constructed within perpendicular axes that intersect at a common reference point, usually zero. The axes are labeled.



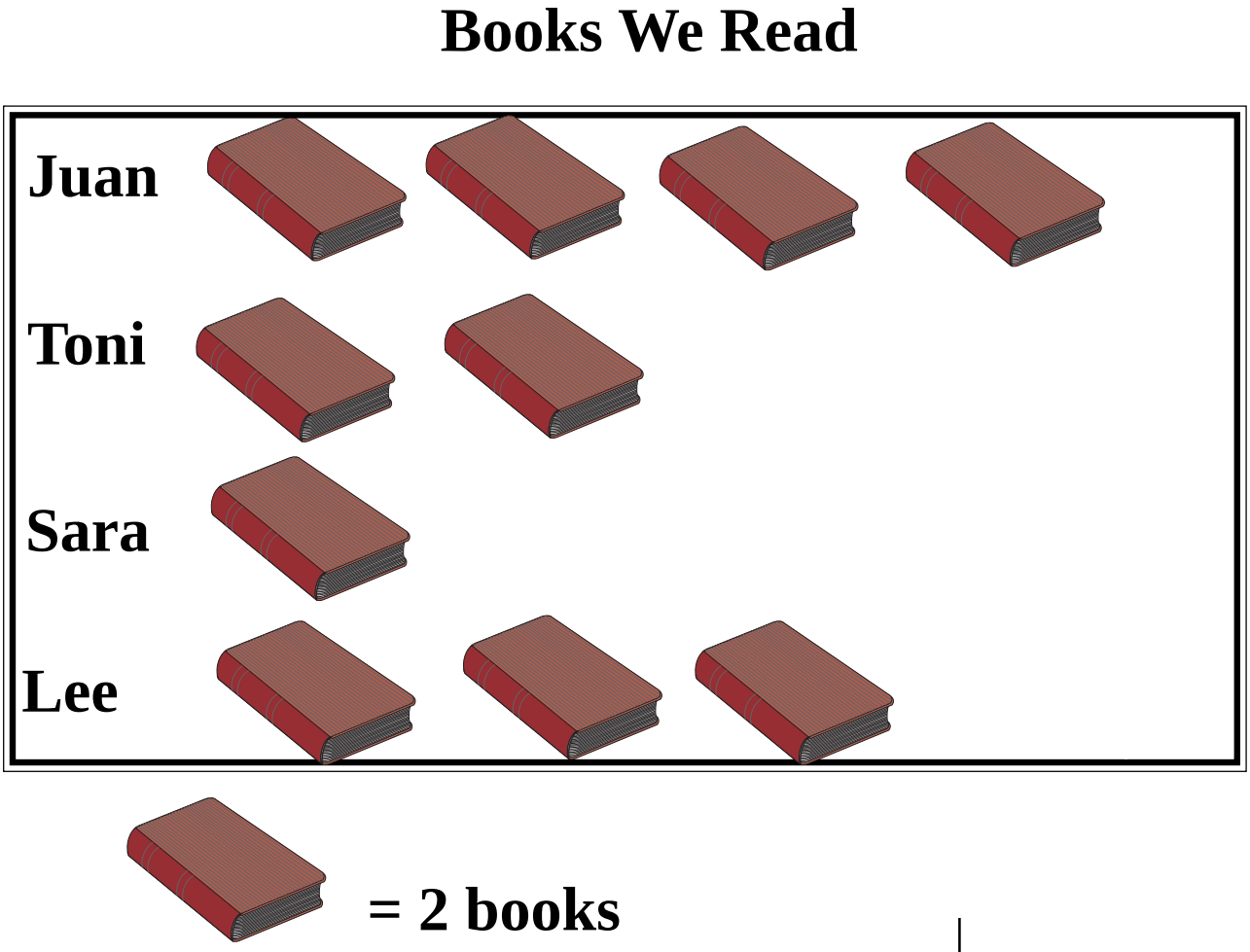
Multiple or double bar graphs are used to compare discrete stratified date (i.e., data collected from particular groups). For example, when asking children to vote for their favorite pets, colors, or favorite games to play, organize the results according to boys' responses and girls' responses.



Picture graphs. Picture graphs (pictographs) use the pictures to depict quantities of objects or people with respect to labeled axes. They are used when the data are discrete (i.e., noncontinuous). The symbols (ideographs) need to be the same size and shape. These symbols may represent real objects (e.g., a stick figure to represent a person or a carton to represent milk drunk by students) or they may take the form of something more abstract (e.g., a triangle or square).

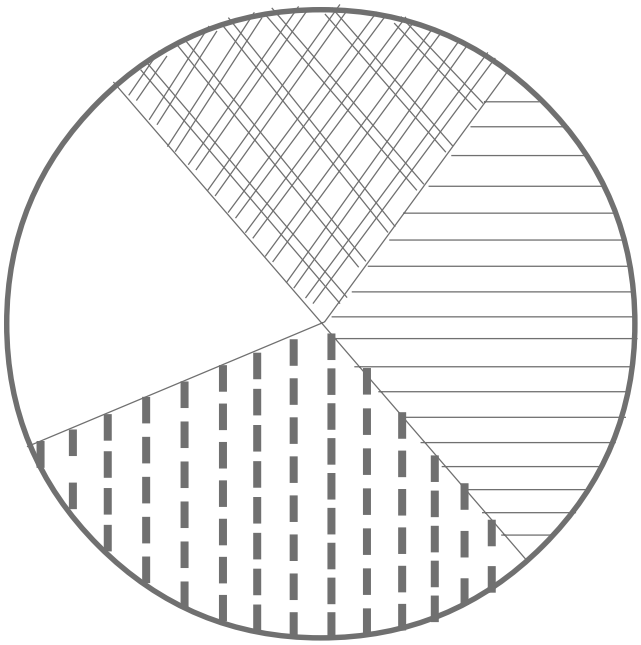
Notes and textbook references

In picture graphs without a legend, the ideograph and the item it represents are in a one-to-one correspondence. When a legend is used, the ratio of each ideograph to the number of objects it represents must be taken into consideration when interpreting the graph. Fractional parts of ideographs (e.g., one-half of a picture) may cause some difficulties for children. Data presented in picture graphs are usually appropriate for bar graphs. Converting picture graphs to bar graphs is one way to help children move from semi-concrete representations of data to more abstract forms.



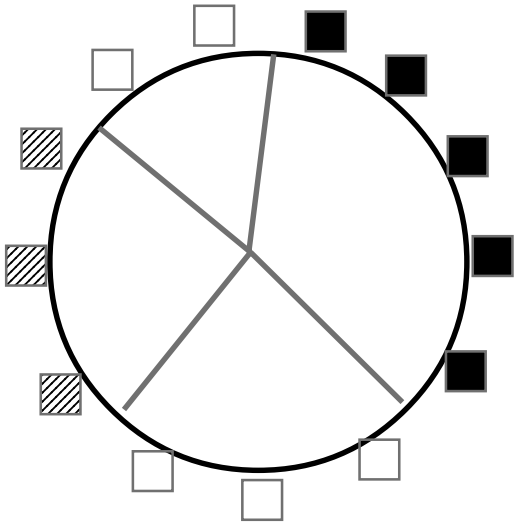
Notes and textbook references

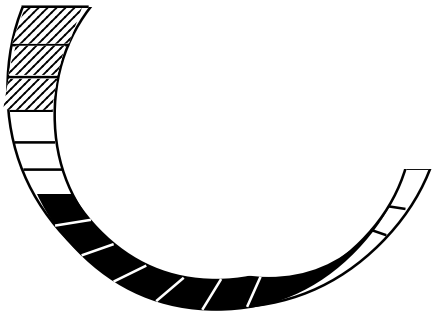
Circle graphs. The area of the circle graph (pie graph, pie chart, area graph) is divided into sections by lines emanating from the center of the circle. Circle graphs are appropriate when children have an understanding of fractions; they provide children with a means of displaying the relationship of parts to whole.



Children may create circle graphs informally before they are able to measure angles and figure proportions. For example, counters representing the total units are evenly spaced around the circle. When the divisions occur, a radius is drawn to divide the circle into appropriate parts. A second informal method is to mark units on a strip and then loop to form a circle, drawing radii as appropriate.

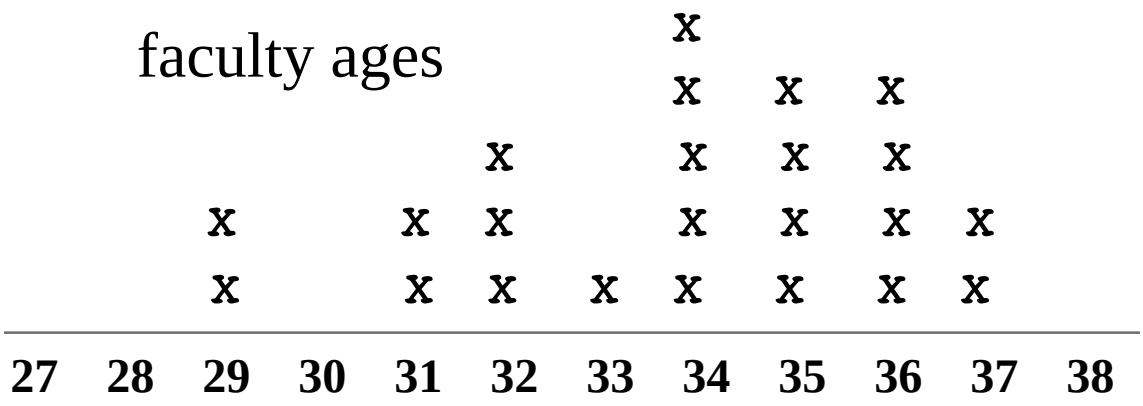
Notes and textbook references





Notes and textbook references

Line plots. Line plots look like primitive bar graphs, where numerical data are plotted as x's placed above numbers on a number line. A line plot "gives a graphical picture of the relative sizes of numbers, and it helps you to make sure that you aren't missing important information" (*Developing Graph Comprehension,* Landwehr and Watkins 1986, p. 5). Unlike a bar graph, in which data may be lost in the grouping, none of the data get lost in a line plot.



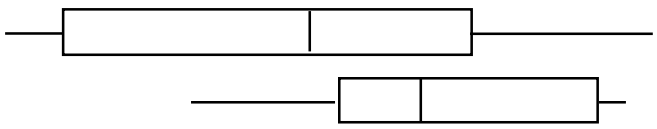
Stem-and-leaf plots. These plots are characterized by a "separation" of the digits in numerical data. For example, in a simple stem-and-leaf plot the tens digits are listed in one column and the ones digit are listed in a row next to the respective tens digit. When rotated ninety degrees counterclockwise, the stem-and-leaf plot resembles a bar graph. The development of the stem-and-leaf plot and the box plot have been attributed to John Tukey (1977).

Students will meet stem-and-leaf plots and box plots in later years.

Spelling Scores

9	5 5 6 7 0
8	7 7 5 4 3 2
7	8 7 5 5 4 4 2 0
6	9 9 8 8 8 1
5	6

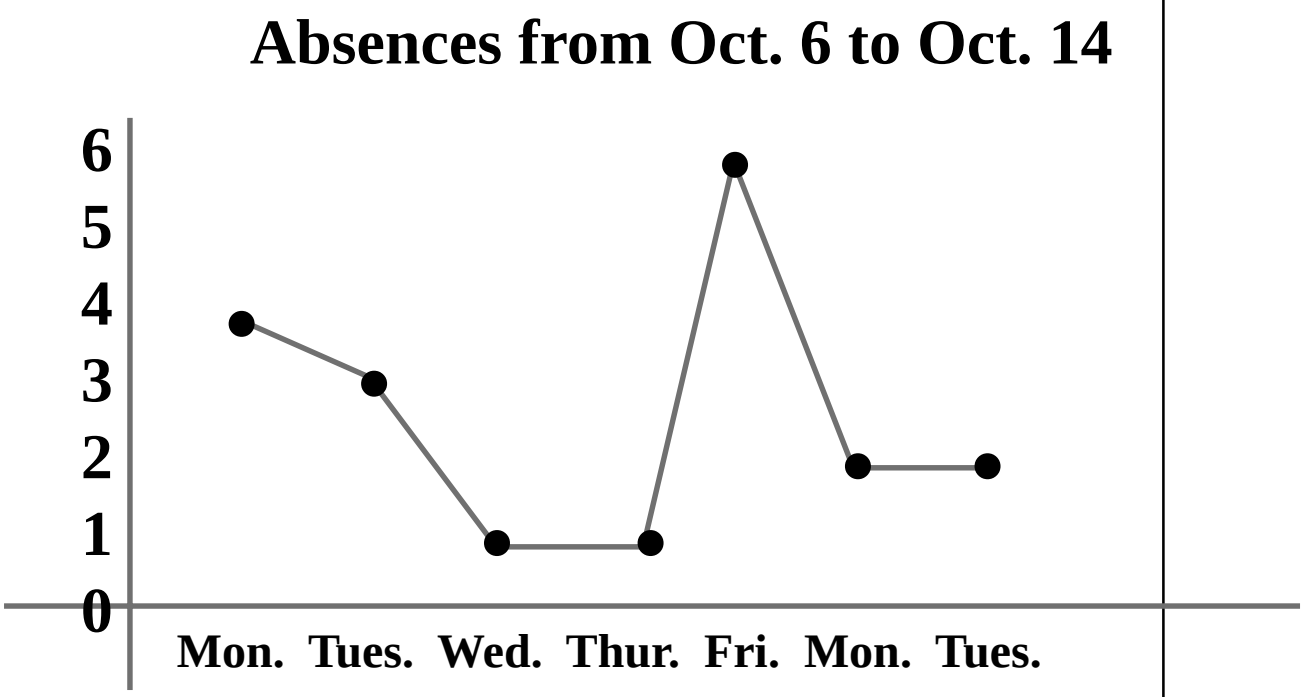
Box plots. Box plots (box-and-whisker plots) use five summary numbers (i.e., the lower extreme, the lower quartile, the median, the upper quartile, and the upper extreme) and are helpful when analyzing large quantities of data (i.e., more than 100 pieces of data). Although this type of display may be more difficult to construct, it has been used effectively with middle school students.



Line graphs. A line graph (broken-line graph) is used to compare continuous data. Points are plotted within perpendicular axes to represent change over a period of time or any linear functional relationship. The labeled axes intersect at a common point, usually zero. The units of division on each axis are equally spaced, and the graphed points are connected by straight or broken lines. When children keep a record over a period of time of their own height or weight, of the daily average temperature, and so on, line graphs are appropriate displays.

Multiple line graphs are used to compare two or more sets of continuous data: for example, to compare the heights or weights of two children over a period of time (e.g., four months or one year), or the heights of two (or more) plants over a period of time (e.g., one to two months after planting seeds).

Notes and textbook references



Notes and textbook references

Levels of Graph Comprehension

While it is important for students to be able to read data presented in graphical forms, a greater potential is realized when they are able to interpret and generalize from the data. Regardless of the graph form used, the three levels of graph comprehension are reading the data, reading between the data, and reading beyond the data.

Reading the Data. This level of comprehension requires a literal reading of the graph. The reader simply "lifts" the facts explicitly stated in the graph or the information found in the graph title and axis labels directly from the graph. There is no interpretation at this level. Reading that requires this type of comprehension is a very low level cognitive task.

Reading between the Data. This level of comprehension includes the interpretation and integration of the data in the graph. It requires the ability to compare quantities (e.g., greater than, tallest, smallest) and the use of other mathematical concepts and skills (e.g., addition, subtraction, multiplication, division) that allow the reader to combine and integrate data, and identify the mathematical relationships expressed in the graph. This is the level of comprehension most often assessed on standardized tests.

Reading beyond the Data. This level of comprehension requires the reader to predict or infer from the data by tapping existing schemata (i.e., background knowledge, knowledge in memory) for information that is neither explicitly nor implicitly stated in the graph. Whereas reading between the data might require that the reader make an inference based on the data presented in the graph, reading beyond the data requires that the inference be made on the basis of a "data base" in the reader's head, not in the graph.

As students experience the physical creation of a graph, they should be involved in interpreting it. Questions that reflect reading the data, reading between the data, and reading beyond the data provide a basis for interpreting and discussing graphs.

Discussions about graphs should revolve around listening, speaking, reading, and writing. Activities that provide children with the opportunity to interpret graphs and plots should include teacher-made and student-formulated questions reflecting different levels of comprehension. Children should be encouraged to write about graphs to clarify their thinking and communicate their interpretations with others. Working in groups of four or five students, they should talk about the graphs they create.