

2008 MATHEMATICS

Standard Course of Study and Grade Level Competencies

K-5

K-5 Transition 2009-2010

K-5 Implementation 2010-11

TABLE OF CONTENTS

ACKNOWLEDGMENTS	1
PREFACE	2
PHILOSOPHY	4
PURPOSE	6
THE MATHEMATICS PROGRAM.....	9
EARLY GRADES K-2	10
Kindergarten	12
Grade 1	15
Grade 2	18
INTERMEDIATE GRADES 3-5	22
Grade 3	24
Grade 4	28
Grade 5	32

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We wish to express special thanks to:

- the Office of Curriculum and Instructional Services for providing the leadership and vision that guided the development of this document,
- the many local educators, higher education partners, parents, and community members who participated in the current revision process by serving on curriculum committees and responding to draft documents, and
- the Department of Public Instruction staff who carried the primary responsibility for revising and editing the *Standard Course of Study*.

The current revision process involved the entire mathematics education community in some way, and its end product is a mathematics curriculum of which North Carolina can be proud.

PREFACE

Intent

The intent of the *North Carolina Mathematics Standard Course of Study* is to provide a set of mathematical competencies for each grade and high school course to ensure rigorous student academic performance standards that are uniform across the state. It is not meant to be an instructional manual. It does not provide strategies for teaching or lesson plans.

Teachers will find NCDPI-developed support documents, more useful in lesson planning and design. These documents will provide more detailed recommendations and support for teaching and assessing the intended curriculum.

The *North Carolina Mathematics Standard Course of Study* clearly defines a curriculum supporting the ABC's school reform effort as well as the North Carolina Testing Program. These revisions maintain a forward focus, looking at what students will need to know and be able to do to be successful and contributing citizens in our state and nation in the years ahead.

Revisions

North Carolina has had a *Standard Course of Study* since 1898. The Basic Education Program was enacted into law in 1985 and called for “a set of competencies by grade level, for each curriculum area.” In 1997 the Excellent Schools Act included the following:

The State Board of Education shall develop a plan to create rigorous student academic performance standards for kindergarten through eighth grade and student academic performance standards for courses in grades 9-12. The performance standards shall align, whenever possible, with the student academic performance standards developed for the National Assessment of Educational Progress (NAEP).

The *North Carolina Mathematics Standard Course of Study* was last revised in 2003. Advisory committee meetings generated discussions centered on initiatives in mathematics education developed or published since 2003. The review included the Trends in International Mathematics and Science Study (TIMSS, 2007) and the Program for International Assessment (PISA, 2006), Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (NCTM, 2006), the National Assessment of Educational Progress Mathematics Framework (NAEP, 2005) and the College Board Standards for College Success: Mathematics and Statistics (2006 and 2007).

The current revisions continue to build upon the work of the North Carolina Mathematics Framework that is “based on a philosophy of the teaching and learning of mathematics that is consistent with the current research, exemplary practices, and national standards.” The primary goal of this document is to provide content requirements that lead students to attain proficiency in mathematics. The objectives set clear, concise, and measurable expectations for all students. This *North Carolina Mathematics Standard Course of Study* provides expectations that students demonstrate competence in conceptual understanding, computing, applying, and reasoning. Classroom activities should encourage students to explore, make conjectures, reason logically, and use a variety of mathematical methods effectively and efficiently to solve problems.

A team of mathematics educators in North Carolina developed the revisions in content for this document.

Program Review

In order to create and maintain a quality program, a continuing re-evaluation of all aspects of the mathematics education program is necessary. There continues to be an urgent need to examine:

- The roles of teachers and students in classrooms;
- The content of school mathematics;
- Assessment practices;
- The preparation and professional development of teachers; and
- The level of support for mathematics education from all parts of society.

PHILOSOPHY

North Carolina public schools have the challenge to provide all students with the mathematical knowledge, skills, and confidence they will need to compete in a technology-oriented workforce and to be informed citizens. With national standards, research in learning, and the increasing role of the federal government in education, there is an emerging consensus about the essential elements of mathematics content and instruction.

The *North Carolina Mathematics Standard Course of Study* is organized in six strands or goals for K-5: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra, and Problem Solving. The objectives for each goal progress in complexity at each grade level. The curriculum has been designed around key ideas that should not be piecemealed into incidental details that address low-level skills. Success in mathematics integrates knowledge, conjecture, and facility with a variety of mathematical concepts. The goal of mathematics instruction should be to produce learners who comprehend concepts, operations, and relationships in mathematics as well as proficiency in computation and the application of those concepts.

The early grades focus on building a strong understanding of number and fluency with mathematics to solve problems. Fundamental to these skills is knowledge of number facts, the computational processes, and the appropriate use of each operation. Together with an emphasis on using mathematics to solve problems, elementary students will build a depth of understanding enabling them to apply the content in a variety of contexts.

Fluency in mathematics is an expectation for all students. Fluency incorporates three ideas: efficiency, accuracy, and flexibility. Students can get bogged down with procedures and calculations that lead to errors. They become efficient as they develop strategies that are manageable, understandable, easily carried out, and generate results that solve problems. Students must develop an accurate knowledge of number facts and number relationships in order to reason and solve problems well. Flexibility is the product of students' successful experiences with problems using a variety of strategies and the analysis of the strategies to determine their efficiency and accuracy.

Mathematics has its own language, and the acquisition of specialized vocabulary and language patterns is critical to a student's understanding and appreciation of the subject. Students need to use correctly the concepts, skills, symbols, and vocabulary identified in the standards set in this document. Students should talk about mathematics and use the language to verify solutions to mathematical problems.

Problem solving and reasoning are stressed throughout the goals at each grade and in every course. The development of problem-solving skills is a major goal of the mathematics program. Experiences in problem-solving processes should permeate instruction. Problem solving should be integrated early and continuously into each student's mathematics education. Students need a wide range of skills and strategies to use as a tool for representing and solving a variety of problems.

Mathematical modeling is an important technique used to build understanding of abstract ideas. Teachers need to expose students to physical representations that help develop understanding of abstract concepts. Early years should include work with manipulatives to help form a sense of number, and work with geometric shapes and patterns facilitates the development of spatial reasoning. In later studies, students will generate algebraic expressions, another form of modeling, which represent physical, social, or natural phenomena and help them make predictions.

One of the challenges facing education today is the development of effective mechanisms for informing teachers about this research so that they can transform the learning environment in their classrooms. Research shows that students develop mathematical competence and power by engaging in solving meaningful problems. Beginning in the earliest grade levels, students should use their own knowledge and experience, working alone, in pairs, and in small and large groups, to solve challenging tasks. They should be expected to communicate their thinking with pictures, numbers and words. Teachers should encourage students to question one another when an explanation doesn't make sense to them. This problem-centered approach to learning mathematics will enable students to take greater responsibility for their own learning, to develop essential communication and decision-making skills, and to understand the fundamental concepts of mathematics, all of which will be critically important to them.

PURPOSE

The vision and philosophy described throughout this document are based on our goals in mathematics education for North Carolina students.

The six goals are for *all* students to develop:

- Strong mathematical problem-solving and reasoning abilities;
- A firm grounding in essential mathematical concepts and skills, including computation and estimation;
- Connections within mathematics and with other disciplines;
- The ability to use appropriate tools including technology to solve mathematical problems;
- The ability to communicate their understanding of mathematics effectively; and
- Positive attitudes and beliefs about mathematics.

These goals for our students in mathematics are the foundation for the rest of the document and guide the development of the critical areas in mathematics education.

Five components have been identified as critical for achieving the goals for our students and as making a significant impact on the quality of mathematics education. A summary of each of these is given here. They are:

- Teaching and Learning
- Content
- Assessment
- Preparation and Professional Development of Teachers
- Roles and Responsibilities

Teachers are the keys to changing the learning environment in North Carolina's classrooms. They plan classroom experiences and create a supportive environment for learning to take place. A teacher plays many roles in today's classrooms as the guide, the coach, the facilitator, and the instigator of mathematical explorations. Through their classroom practices, teachers promote students' mathematical reasoning, challenge them with rich problems through which they learn to value mathematics, and provide them with a strong foundation for further study. Most of all, teachers encourage and support their students' development of mathematical power.

The heart of mathematics is reasoning, which, together with knowledge of appropriate mathematical content, forms the basis of mathematical power. The goal of mathematics education in North Carolina is to enable all students to develop mathematical power and confidence in their ability to do mathematics. A curriculum that focuses on significant mathematical ideas, instead of isolated topics, encompasses both concepts and skills through rich explorations, problems, and applications that enable students to develop a genuine understanding of the big ideas of mathematics.

Assessment is a process of planning, gathering evidence, interpreting evidence, and making decisions. Mathematics assessment is directly related to instruction and student thinking. Four

purposes of assessment are monitoring student progress, making instructional decisions, evaluating student achievement, and evaluating a program.

Assessment has taken on a broader meaning. Beyond grading students, assessment should probe beneath right answers to discover how students think and how instruction can be improved. In this view of assessment, expected outcomes are set and the time necessary for each student to achieve the intended outcomes varies. Every student is challenged to meet a higher standard.

Effective assessment fosters the development of mathematical power. Students must be given opportunities to acquire and demonstrate understanding and depth of knowledge. The criteria for judging mathematical accomplishment must be made public and communicated clearly to students, parents, and other appropriate parties.

Teaching in a manner that cultivates mathematical power for all students is a complex and demanding process that requires intensive lifelong learning. Teachers must have not only extensive knowledge of mathematics but also deep understanding of how students learn mathematics. Appropriate content and pedagogical preparation enables teachers to design lessons and implement curriculum using suitable strategies and resources in an environment where all students have an opportunity to succeed. Teachers are role models for their students, as learners and problem solvers who value and enjoy mathematics. Programs for teachers at all stages in their career must provide them with the tools to implement the goals previously stated.

The professional development of a mathematics teacher is an ongoing process. This development occurs in three distinct phases: pre-service (undergraduate or teacher preparation); induction (the first three to five years of teaching); and in-service (continued professional growth over the span of the teacher's career). Communication and articulation efforts to link these phases are necessary for a continuum of professional growth. Although teachers need to take a major responsibility for their own professional development, it also requires leadership, resources, financial support, and advocacy at the state, district, school, classroom, and university levels.

Many different constituencies in North Carolina must work together to establish a mathematics education program that enables all students in the state to develop their full potential as powerful and creative thinkers and problem solvers.

Support for mathematics education requires that:

- Legislators and other financial partners provide funding that increases the likelihood of student achievement;
- School boards and administrators enact policies which enable teachers to provide quality instruction;
- Colleges and universities help teachers develop content knowledge and expertise in teaching;
- Parents and other citizens become partners with schools to value and nurture student efforts; and
- Educators and students work cooperatively to establish and reach high educational goals.

The education of students is our shared responsibility. All citizens of North Carolina must assume this responsibility and become active advocates for improved mathematics education.

THE MATHEMATICS PROGRAM

The competency goals and objectives of the *North Carolina Standard Course of Study of Mathematics K-5* are organized into six strands: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra and Problem Solving. These strands are not meant to be a sequential guide for instruction but rather an organization of similar objectives under a common topic.

The mathematics program is designed in grade spans that parallel the developmental stages of students: grades K-2, grades 3-5, grades 6-8, and grades 9-12. The elementary program focuses on students actively engaged in the development of mathematical understanding by using manipulatives, working independently and cooperatively to solve problems, and conducting investigations and recording findings. Middle grade students expand their skills to compute with all real numbers and are challenged to apply their prior knowledge and experience in new and more difficult situations. The core high school mathematics program includes a choice of the integrated sequence of courses (Integrated Mathematics 1, 2, 3) or the conventional sequence of courses (Algebra 1, Geometry, Algebra 2) plus one additional mathematics course to complete a minimum of 4 mathematics credits for graduation from high school. Additional courses outside the core courses are intended to offer opportunities that address the needs of individual students.

Elementary School Mathematics		
Middle School Mathematics		
High School Courses (Core Course of Study)		
Integrated Mathematics 1	OR	Algebra 1
Integrated Mathematics 2	OR	Geometry
Integrated Mathematics 3	OR	Algebra 2
Integrated Mathematics 4	OR	Pre-Calculus
Discrete Mathematics	OR	Advanced Functions and Modeling
High School Courses (Advanced Placement Courses)		
Advanced Placement Statistics		Advanced Placement Calculus

The primary difference in the 2008 revision of the *North Carolina Mathematics Standard Course of Study K-5* is not the addition of new objectives, but the increased depth and complexity of the objectives. In addition, a problem-solving strand has been added. The problem solving strand illustrates the connections among mathematical ideas and illuminates the reasoning and communication skills necessary for success in the 21st Century.

EARLY GRADES K-2

Number and Operations

The concepts and skills related to number and operations are essential to the mathematics instruction during the primary years. It is during these early years when students' understanding of number develops considerably. In the primary grades, students learn how to count sets, describe patterns in counting and make predictions on those patterns, and read and write whole numbers. Students recognize different representations for whole numbers and are able to explain why those representations are equivalent. During the kindergarten through second grade years, students should explore, observe and reflect on further questions about whole numbers such as, "How many different ways can you make 10? Which numbers can you put into groups of 4?" and so forth. In the early grades, students represent whole numbers concretely, pictorially and symbolically. Whole numbers are compared and ordered and students use a variety of strategies to estimate quantities and understand place value. Students develop multiple strategies to solve joining and separating problems. They become fluent with addition and subtraction facts to 18.

Measurement

Students in the early grades use a variety of materials to begin to understand the processes and components of measurement. As they learn about different tools for measuring, they describe, estimate, compare and measure using non-standard and standard units. Through many experiences and opportunities to explore, students identify the need for standard measurement. Students use the calendar to follow the days of the week, months of the year and the cycle of time.

Geometry

Kindergarten through second grade students learn the names and use basic properties to describe, compare and sort simple geometric shapes. They use spatial visualization skills to cut or rearrange shapes to form new shapes. Early grade students begin to see the relationship of the two dimensional shapes to the faces of three dimensional shapes. As students become familiar with the geometric shapes, they begin to look for the shapes in their environment and practice replicating and using the shapes. They also learn the meaning of basic directional and positional relationships.

Data Analysis

Students are introduced to the process of statistical investigation. They pose questions that are relevant to their world and then collect data to answer the questions. Students may do this by counting, measuring, and conducting simple surveys and experiments. They organize, describe, analyze and display their data.

Algebra

In the early grades, students learn about patterns and describe objects by their attributes. They compare, sort and order objects by one or more characteristics. Their understanding is extended by finding and creating patterns, correcting errors in patterns and translating patterns into different forms. Students understand that a numeral is a symbol for a quantity. They explore the use of variables and write open sentences to express relationships. A major concept at this level is that students understand equality.

Problem Solving

Students in the primary grades see the connections between the mathematical ideas. They use their mathematical reasoning to discuss, reflect on and justify this reasoning. By using concrete materials and appropriate technologies, such as calculators and computers, students actively participate in their learning of mathematics. The goal is for students to develop efficient strategies that make sense to them and to solve problems accurately.

Kindergarten

Major Concepts/Skills

- Number sense 0 – 10
- Part-part whole and equality relationships
- Repeating patterns
- Sort and classify
- Cyclical nature of time
- Reason and communicate mathematical ideas

Strands: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra, Problem Solving and Reasoning

COMPETENCY GOAL 1:

The learner will understand numbers and ways to represent numbers.

Objectives

1.01 Develop number sense for whole numbers from 0 through 10.

- a) Use 1 to 1 correspondence and other counting strategies to determine how many.
- b) Connect the model, number word (orally) and numeral using multiple representations.
- c) Compare and order sets and numbers (more than, less than, same/equal).
- d) Recognize and write numerals.
- e) Recognize (subitize) the amount in a given set of patterned dots/objects.
- f) Determine number before and after a given number.
- g) Identify and sequence ordinal numbers.
- h) Compose and decompose numbers:
 - recognize part-part-whole relationships.
 - use 5 and 10 as referents.

1.02 Model and count objects in a set and rote count:

- a) forward to 30.
- b) backward from 10.

1.03 Demonstrate and illustrate the meaning of joining and separating sets with objects to solve problems for whole numbers from 0-10; use informal language to describe the strategies.

COMPETENCY GOAL 2:**The learner will investigate the concepts and processes of measurement.****Objectives**

- 2.01 Recognize length and mass as measurable attributes.
- 2.02 Compare and order objects with respect to a given attribute (length, mass).
- 2.03 Make connections between events and experiences and the cyclical measure of time.
 - a) Sequence common events.
 - b) Sequence days of the week, months, and seasons.

COMPETENCY GOAL 3:**The learner will investigate the concepts of geometry.****Objectives**

- 3.01 Identify, describe, compare, and sort geometric three-dimensional shapes (spheres, cubes, cylinders, and cones) by attributes.
- 3.02 Identify, model, describe, compare, and sort geometric two-dimensional shapes (triangles, rectangles including squares, and circles) by attributes.
- 3.03 Demonstrate spatial reasoning to fill shapes and model objects found in the environment.
- 3.04 Identify objects in the environment and describe their relative locations using positional and directional words.

COMPETENCY GOAL 4:**The learner will collect, organize, and display data to answer questions.****Objectives**

- 4.01 Use the process of statistical investigation as a group activity.
 - a) Pose questions and collect data about themselves and their surroundings.
 - b) Organize and represent data using concrete objects, pictures or pictorial graphs.
 - c) Evaluate how the data help answer the posed question.

COMPETENCY GOAL 5:

The learner will investigate algebraic concepts including object classification, patterns and equality.

Objectives

- 5.01 Identify attributes, sort objects by one attribute and justify the rule used to classify.
- 5.02 Identify, duplicate, extend, and create repeating patterns using actions, words and models.
- 5.03 Develop an understanding of the relationship between part-part-whole and the concept of equality.

COMPETENCY GOAL 6:

The learner will make connections, solve problems and reason mathematically.

Objectives

- 6.01 Recognize and apply connections among mathematical ideas to solve problems.
- 6.02 Develop fluency in solving problems that arise in mathematics and in other contexts, building mathematical knowledge through problem solving.
- 6.03 Use reasoning to understand situations and extend thinking.
- 6.04 Use the language of mathematics and appropriate technology to communicate mathematical ideas demonstrating understanding of problems and solutions.
- 6.05 Explain personal representations that communicate mathematical ideas.

Grade 1

Major Concepts/Skills

- Number sense to at least 100
- Counting sense
- Addition and subtraction
- Equivalence
- Non-standard measurement
- Collect and display data
- Create, extend and translate patterns

Concepts/Skills to Maintain

- Basic geometric shapes
- Sort and classify

Strands: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra, Problem Solving

COMPETENCY GOAL 1:

The learner will demonstrate an understanding of the relationships among numbers, operations, and equality.

Objectives

- 1.01 Develop number sense for whole numbers to at least 100.
 - a) Count, read and write numbers.
 - b) Connect the model, number words and numbers using a variety of physical models and other representations.
 - c) Describe patterns in counting by ones (both forward and backward) and skip counting; use those patterns to predict the number before and after the counting sequences.
 - d) Use efficient counting strategies to determine how many; explore groupings of objects by 2's, 5's, 10's to determine how many.
 - e) Compose and decompose numbers (part-part-whole).
 - f) Recognize equivalence; compare and order sets and numbers using words rather than symbolic notation.
- 1.02 Represent whole numbers greater than 10 in groups of tens and ones using objects, pictures, and numbers.
- 1.03 Develop fluency with recall of addition facts (sums to 10) and the related subtraction facts.
- 1.04 Demonstrate the meaning and use of addition and subtraction to 100 *with models*.
 - a) Explore properties of addition (i.e. commutative and identity) and the inverse relationship between addition and subtraction.

- b) Model, describe, and illustrate these operations in contexts using multiple strategies.
- c) Estimate sums and differences and justify the reasonableness of solutions in meaningful contexts.

COMPETENCY GOAL 2:

The learner will develop an understanding of the process of measuring using nonstandard units.

Objectives

- 2.01 Compare and order two or more objects or events with respect to a given attribute (length, mass, capacity, duration).
- 2.02 Select appropriate non-standard units to estimate and measure objects with respect to a given attribute using multiple copies of same-size units.

COMPETENCY GOAL 3:

The learner will analyze characteristics properties and relationships of geometric shapes.

Objectives

- 3.01 Identify, describe, compare, and sort geometric three-dimensional shapes (including prisms and pyramids) by attributes.
- 3.02 Identify, model, describe, compare, and sort geometric two-dimensional shapes (including parallelograms, rhombus, trapezoids, and hexagons) by attributes.
- 3.03 Predict and investigate the results of composing and decomposing two-dimensional shapes and describe the part-whole relationships of these shapes.
- 3.04 Describe relationships of objects using proximity, position and direction; follow and/or formulate directions to move or place an object.

COMPETENCY GOAL 4:

The learner will gather, organize, and analyze data to answer questions.

Objectives

- 4.01 Use the process of statistical investigation.
 - a) Pose questions, collect data to answer questions, and make decisions using data.
 - b) Organize and represent data using concrete objects, pictures or pictorial graphs, line plots and tallies.
 - c) Describe data in a variety of ways and evaluate how the data help answer the posed question.

COMPETENCY GOAL 5:**The learner will analyze patterns and simple mathematical relationships.****Objectives**

- 5.01 Sort objects by two attributes; identify and justify the rule used to classify.
- 5.02 Analyze, correct errors in, extend and translate repeating patterns into different forms using actions, words, and models.
- 5.03 Model, write, and evaluate simple number sentences (equations) to develop an understanding of equality.
- 5.04 Determine and justify the value of the unknown in simple number sentences using models and symbols.

COMPETENCY GOAL 6:**The learner will make connections, solve problems and reason mathematically.****Objectives**

- 6.01 Recognize and apply connections among mathematical ideas.
 - a) Connect concepts and skills from previous years to current objectives.
 - b) Connect concepts and skills from multiple strands to solve problems.
- 6.02 Develop fluency in solving single and multi-step problems that arise in mathematics and in other contexts, building mathematical knowledge through problem solving.
- 6.03 Use reasoning to solve problems.
 - a) Understand situations and communicate mathematical problem solving.
 - b) Make estimates with appropriate ranges.
 - c) Reflect, extend and refine thinking.
- 6.04 Use the language and symbols of mathematics and appropriate technology to:
 - a) solve problems;
 - b) communicate mathematical ideas;
 - c) demonstrate understanding of problems and solutions through oral, pictorial, and written explanations.
- 6.05 Create and use representations to organize, record and communicate mathematical ideas.

Grade 2

Major Concepts/Skills

- Number sense to at least 1000
- Place value
- Addition and subtraction of multi-digit numbers
- Length
- Venn diagrams
- Pictographs
- Equations; solve for unknown
- Money
- Number sequences

Concepts/Skills to Maintain

- Repeating patterns
- Sort and classify
- Line plots, tallies

Strands: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra, Problem Solving and Reasoning

COMPETENCY GOAL 1:

The learner will develop an understanding of place value and compute with whole numbers.

Objectives

- 1.01 Develop number sense for whole numbers to at least 1,000.
 - a) Connect the model, number words, and numbers using a variety of physical models and other representations.
 - b) Identify, describe and construct multiple representations.
 - c) Compose and decompose numbers (part-part-whole).
 - d) Read and write numbers.
 - e) Compare and order (including the use of symbolic notation).
 - f) Use a variety of models to build understanding of place value.
 - g) Use 10's and 100's as units for counting, increasing and decreasing quantities by 10's and 100's from any given number.
- 1.02 Develop fluency with recall of addition facts (sums to 18) and the related subtraction facts.
- 1.03 Solve problems involving multi-digit addition and subtraction situations.
 - a) Understand and use properties of addition (i.e. commutative, associative, and identity) and the inverse relationship between addition and subtraction.
 - b) Explore multiple ways of solving multi-digit problems.
 - c) Develop efficient strategies for computing.

d) Estimate sums and differences and justify the reasonableness of solutions in meaningful contexts.

1.04 Create, model, and solve problems that use fair shares (between two, three, or four).

COMPETENCY GOAL 2:

The learner will apply the processes and components of measuring using nonstandard and standard units.

Objectives

2.01 Use non-standard units to develop an understanding of processes for measuring (length, mass, and capacity) recognizing that:

- a) the type of unit used to measure depends on the attribute being measured,
- b) larger units can be subdivided into equivalent units (partitioning),
- c) the same unit can be repeated to determine the measure (iteration), and
- d) the relationship between the size of the unit and the number of units needed (compensatory principle).

2.02 Select and use appropriate non-standard units and standard units (inches and feet) to estimate length, develop and use personal benchmarks (referents) for length, and measure length to the nearest whole unit.

2.03 Develop a sense of intervals of time.

2.04 Recognize coins (penny, nickel, dime, quarter) and compare the value of each; create sets and find the value of a group of coins up to 99 cents.

COMPETENCY GOAL 3:

The learner will recognize and use the basic properties of basic two- and three-dimensional shapes.

Objectives

3.01 Describe attributes and construct three-dimensional shapes; relate the shapes of the faces of three-dimensional objects to two-dimensional shapes.

3.02 Describe, sort, and create congruent shapes.

- a) Determine whether shapes are congruent.
- b) Recognize congruency in shapes with different orientations.

COMPETENCY GOAL 4:**The learner will demonstrate an understanding of and apply the statistical process.****Objectives**

- 4.01 Use the process of statistical investigation.
- a) Pose questions and collect data to answer questions.
 - b) Organize, represent and compare data using various representations including Venn diagrams, pictographs, tallies and line plots.
 - c) Describe parts of data using counting, concepts of grouping and comparing to illustrate the differences between values and frequencies.
 - d) Identify patterns and trends to make decisions using data.

COMPETENCY GOAL 5:**The learner will demonstrate an understanding of equality and number sequences.****Objectives**

- 5.01 Represent, describe, find missing terms, and extend nonnumeric repeating and growing patterns.
- 5.02 Develop fluency with arithmetic sequences to build knowledge of:
- a) odd and even numbers;
 - b) number sequences that grow by 2's, 5's, 10's;
 - c) number sequences that increase and decrease by 10's from any given number.
- 5.03 Model, write and evaluate addition and subtraction number sentences (equations).
- a) Represent a problem including using symbols to represent unknown quantities.
 - b) Demonstrate an understanding of equality to find the value of unknown quantity.

COMPETENCY GOAL 6:**The learner will make connections, solve problems and reason mathematically.****Objectives**

- 6.01 Recognize and apply connections among mathematical ideas.
- a) Connect concepts and skills from previous years to current objectives.
 - b) Connect concepts and skills from multiple strands to solve problems.
- 6.02 Develop fluency in solving single and multi-step problems that arise in mathematics and in other contexts, building mathematical knowledge through problem solving.

6.03 Use reasoning to solve problems.

- a) Understand situations and communicate mathematical problem solving.
- b) Make estimates with appropriate ranges.
- c) Reflect, extend and refine thinking.

6.04 Use the language and symbols of mathematics and appropriate technology to:

- a) solve problems;
- b) communicate mathematical ideas;
- c) demonstrate understanding of problems and solutions through oral, pictorial, and written explanations.

6.05 Create and use representations to organize, record and communicate mathematical ideas.

INTERMEDIATE GRADES 3-5

Number and Operations

Students in the intermediate grades represent whole numbers, fractions, and decimals with concrete objects, pictures, and symbols in a variety of contexts. A firm understanding and use of the place value system and various properties of numbers is developed. Students recognize equivalent rational numbers and explain the basis for the equivalence. They understand the relationship between fractions and decimals. Students at this level compare and order whole numbers, fractions and decimals. At this level, students develop fluency with all whole number operations and with addition and subtraction of fractions and decimals. Students learn the order of operations, explore various properties of operations, and are able to estimate reasonable answers.

Measurement

Third through fifth grade students understand the processes and components of measurement. They understand unit iteration, transitivity, and that the size of the unit affects the number of units needed. Students at this grade span understand the need for standard measurement. Students estimate and measure temperature, length, mass, and capacity in both customary and metric units. They solve conversion problems within the same system of measurement (i.e. inches to feet or centimeters to meters). Students at this level solve problems involving perimeter of plane figures and area of rectangles and develop the basic formulas for computing these quantities.

Geometry

In the intermediate grades, students compare, describe, classify, and analyze two- and three- dimensional shapes. They investigate geometric relationships and properties, including angles, parallelism, perpendicularity and geometric transformations. In this grade span, students work with both line and rotational symmetry. Students understand and use the rectangular grid.

Data Analysis and Probability

Students continue working with the process of statistical investigation and the techniques for data collection become more sophisticated. Third through fifth grade students begin understanding the measures of central tendency and the variability of data sets. Data are described and compared using median, mode, and range. They use this information to make predictions or conclusions about the data set. Students perform simple probability experiments.

Algebra

Students in this grade span build on their understanding of growing patterns and relationships. They use tables and graphs to show relationships, extend data, make predictions and generalize rules. Students use rules to describe functional relationships in arithmetic sequences. At this level, students demonstrate their understanding of equality by evaluating equations, representing problems using variables, and finding the value of variables. Equality continues to remain a major component of the Algebra strand at this level.

Problem Solving

As students deepen their mathematical understanding, they are able to make connections between concepts. They develop fluency in solving single and multi-digit problems that arise in mathematics and in other contexts. Students at this level use mathematical vocabulary as they model, reflect on, explain and justify their thinking. They use appropriate problem solving strategies and technology to organize and communicate their ideas.

Grade 3

Major Concepts/Skills

- Number sense at least to 10,000
- Concepts of multiplication and division
- Fractions
- Customary system of measurement
- Coordinate grids
- Bar graphs, tables
- Simple Probability
- Growing patterns
- Variables
- Time

Concepts/Skills to Maintain

- Congruence
- Line plots, tallies, pictographs
- Venn diagrams
- Repeating patterns
- Odd and even
- Number sequences
- Three dimensional shapes
- Sort and classify

Strands: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra, Problem Solving and Reasoning

COMPETENCY GOAL 1:

The learner will demonstrate an understanding of fractions and whole number operations.

Objectives

- 1.01 Develop number sense for rational numbers to at least 10,000.
 - a) Demonstrate multiple ways to represent numbers using models, words and symbolic representations.
 - b) Identify the place and the value of a given digit in order to determine the magnitude of the number.
 - c) Compare and order (including the use of symbolic notation).
- 1.02 Develop understanding of the part-whole meaning of fractions as sharing equally with area, set, region, and length models.
 - a) Use models and benchmarks (0, $\frac{1}{2}$, 1) to compare and order fractions including common equivalents.
 - b) Model and describe common equivalents among:
 - halves, fourths, and eighths;
 - thirds and sixths.
- 1.03 Develop fluency and flexibility with multi-digit addition and subtraction.
 - a) Use strategies for adding and subtracting numbers (including, but not limited to, standard algorithms)
 - b) Estimate sums and differences and justify the reasonableness of solutions in meaningful contexts.
 - c) Analyze the relationships between operations.

- 1.04 Demonstrate conceptual understanding of the meaning of multiplication and division through multiple models.
- a) Make connections about the multiples and factors of a given number.
 - b) Analyze the relationship between multiplication and division.
- 1.05 Develop fluency with multiplication facts for 1's, 2's, 5's, 10's, 0's and strategies for 3's, 4's, 6's, 7's, 8's, 9's; and related division facts.

COMPETENCY GOAL 2:

The learner will apply the processes and components of measuring using customary measurement units.

Objectives

- 2.01 Develop an understanding of and use the processes for measuring with customary units of measurement (length, weight, capacity, and temperature) recognizing that:
- a) the type of unit used to measure depends on the attribute being measured,
 - b) larger units can be subdivided into equivalent units (partitioning),
 - c) two objects can be compared in terms of a measurable quality using a third object (transitivity),
 - d) the same unit can be repeated to determine the measure (iteration), and
 - e) the relationship between the size of the unit and the number of units needed (compensatory principle).
- 2.02 Develop and use personal benchmarks (referents) for customary measurements to estimate length, weight, capacity, time, and temperature.
- 2.03 Select attributes and appropriate standard units and tools (customary) to estimate and measure length, weight, capacity, temperature, and time to the minute.
- 2.04 Determine the amount of money needed to make change (up to a dollar) using various strategies.

COMPETENCY GOAL 3:

The learner will use the rectangular coordinate system and the basic geometric properties of two-dimensional shapes.

Objectives

- 3.01 Describe, analyze, compare and classify two-dimensional shapes by properties including sides and angles (acute, obtuse, right).
- 3.02 Use rectangular coordinate system to:
- a) graph and identify points with whole number or letter coordinates,
 - b) describe possible paths between given points on the plane,
 - c) identify parallel and perpendicular lines, and
 - d) construct geometric shapes with vertices at points on a coordinate grid.

COMPETENCY GOAL 4:

The learner will use and understand the statistical process and simple probability concepts.

Objectives

- 4.01 Use the process of statistical investigation.
 - a) Pose questions that involve collecting categorical and numerical data.
 - b) Design investigations to answer questions using observations, surveys and experiments.
 - c) Collect, organize, represent and analyze data using various representations including tables and bar graphs.
 - d) Describe the shape of set of data and important features, including concepts of mode and variability (minimum and maximum values and range).

- 4.02 Understand situations involving simple probability.
 - a) Judge the probability of events as being (certain, likely, equally likely, unlikely, possible, or impossible) to occur.
 - b) Conduct simple probability experiments.
 - c) Describe results using pictures and words, and make predictions.

COMPETENCY GOAL 5:

The learner will explore functional relationships and use variables.

Objectives

- 5.01 Analyze numeric and nonnumeric growing patterns to explore functional relationships.

- 5.02 Model, write and evaluate simple multiplication and division equations.
 - a) Represent a problem including using variables to represent unknown quantities.
 - b) Demonstrate an understanding of equality.
 - c) Find the value of variables.

- 5.03 Demonstrate an understanding of the commutative and identity properties for addition and multiplication.

COMPETENCY GOAL 6:

The learner will make connections, solve problems and reason mathematically.

Objectives

- 6.01 Recognize and apply connections among mathematical ideas.
 - a) Connect concepts and skills from previous years to current objectives.
 - b) Connect concepts and skills from multiple strands to solve problems.

- 6.02 Develop fluency in solving single and multi-step problems that arise in mathematics and in other contexts, building mathematical knowledge through problem solving.
- 6.03 Use reasoning to solve problems.
- a) Understand situations and communicate mathematical problem solving.
 - b) Make estimates with appropriate ranges.
 - c) Reflect, extend and refine thinking.
- 6.04 Use the language and symbols of mathematics and appropriate technology to:
- a) solve problems;
 - b) communicate mathematical ideas;
 - c) demonstrate understanding of problems and solutions through oral, pictorial, and written explanations.
- 6.05 Create and use representations to organize, record and communicate mathematical ideas.

Grade 4

Major Concepts/Skills

- Number sense 0.01-to at least 100,000
- Fraction/decimal relationships
- Multiplication and division of multi-digit whole numbers
- Addition and subtraction of decimals and fractions with like denominators
- Perimeter and area
- Transformations
- Line graphs, circle graphs
- Median, mode, and range
- Data distributions
- Variables in number sentences
- Metric Measurement
- Simple (one step) conversions with the same system of measurement
- Simple probability

Concepts/Skills to Maintain

- Whole number computation
- Coordinate grids
- All graphs from previous grades including Venn diagrams
- Customary measurement

Strands: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra, Problem Solving and Reasoning

COMPETENCY GOAL 1:

The learner will build an understanding of and compute with non-negative rational numbers (.01 to at least 100,000).

Objectives

- 1.01 Develop number sense for rational numbers from .01 to at least 100,000.
 - a) Demonstrate multiple ways to represent numbers using models, words and symbolic representations.
 - b) Identify the place and the value of a given digit in order to determine the magnitude of the number.
 - c) Compare and order (including the use of symbolic notation).
- 1.02 Develop understanding of the meanings and uses of fractions and decimals.
 - a) Use models, benchmarks (0, $\frac{1}{2}$, 1, 1.5, 2 and so on), and reasoning to compare and order fractions and decimals.
 - b) Model and describe common equivalents among:

- halves, fourths, eighths, and mixed numbers;
- thirds, sixths, twelfths, and mixed numbers;
- fifths, tenths, hundredths, and mixed numbers.
- c) Understand and use mixed numbers and their equivalent fraction forms.
- d) Make connections between fractions and decimals.

- 1.03 Develop fluency and flexibility with multiplication and division involving:
- a) tables 0-12;
 - b) up to two-digit by one-digit multiplication;
 - c) strategies for two-digit by two-digit multiplication (larger numbers with calculator);
 - d) up to three-digit by one-digit division with and without remainders (larger numbers with calculator);
 - e) estimation of products and quotients and justification of the reasonableness of solutions in meaningful contexts.
- 1.04 Develop fluency with addition and subtraction of decimals and fractions with like denominators.
- a) Develop and analyze strategies for adding and subtracting numbers.
 - b) Estimate sums and differences and justify the reasonableness of the solutions in meaningful contexts.

COMPETENCY GOAL 2:

The learner will apply the processes and components of measurement using metric units and make simple conversions within the same system (e.g. metric to metric or customary to customary).

Objectives

- 2.01 Develop an understanding of and use the processes for measuring with metric units of measurement (length, mass, capacity, temperature) recognizing that:
- a) the type of unit used to measure depends on the attribute being measured,
 - b) larger units can be subdivided into equivalent units (partitioning),
 - c) two objects can be compared in terms of a measurable quality using a third object (transitivity),
 - d) the same unit can be repeated to determine the measure (iteration), and
 - e) the relationship between the size of the unit and the number of units needed (compensatory principle).
- 2.02 Develop and use personal benchmarks (referents) for metric measurements to estimate length, mass, capacity, and temperature.
- 2.03 Select attributes and appropriate standard units and tools (metric) to estimate and measure length, mass, capacity, and temperature.
- 2.04 Make simple unit conversions within the *same* measurement system (metric and customary).

COMPETENCY GOAL 3:

The learner will demonstrate an understanding of symmetry, transformations, area and perimeter.

Objectives

- 3.01 Identify and describe symmetry in two-dimensional shapes; create symmetrical shapes with line and/or rotational symmetry.

- 3.02 Identify, predict, and describe the results of transformations of two-dimensional shapes using reflections, translations, and rotations.

- 3.03 Solve problems involving area and perimeter.
 - a) Cover regions using a variety of objects.
 - b) Create physical and pictorial models of area with and without grids.
 - c) Estimate and measure area of rectangles.
 - d) Estimate and measure perimeter of two-dimensional shapes.
 - e) Explore relationships between area and perimeter.

COMPETENCY GOAL 4:

The learner will understand and use graphs, probability and data analysis.

Objectives

- 4.01 Use the process of statistical investigation.
 - a) Pose questions and design investigations that involve comparing two sets of related data each represented on the same type of graph using the same scale.
 - b) Collect, organize, analyze and display data using various representations including line graphs.
 - c) Analyze data presented in graphs, including circle graphs.
 - d) Compare two distributions of data, including their shapes, measures of center (mode, median) and variability (minimum and maximum values, unusual data points, and range).
- 4.02 Understand situations involving simple probability.
 - a) Determine probability of an event from a context that includes a visual representation.
 - b) List all possible outcomes (sample space) of a situation or an event.

COMPETENCY GOAL 5:

The learner will demonstrate an understanding of mathematical relationships.

Objectives

- 5.01 Analyze nonnumeric and numeric growing patterns.
 - a) Use rules to describe these patterns as functional relationships (arithmetic sequences only).
 - b) Create, extend, and find missing terms.

- 5.02 Model, write and evaluate whole number equations.
 - a) Solve problems, including using variables to represent unknown quantities.
 - b) Demonstrate an understanding of equality and simple inequality.
 - c) Find the value of variables.

- 5.03 Develop an understanding of and apply order of operations in meaningful contexts.

COMPETENCY GOAL 6:

The learner will make connections, solve problems and reason mathematically.

Objectives

- 6.01 Recognize and apply connections among mathematical ideas.
 - a) Connect concepts and skills from previous years to current objectives.
 - b) Connect concepts and skills from multiple strands to solve problems.

- 6.02 Develop fluency in solving single and multi-step problems that arise in mathematics and in other contexts, building mathematical knowledge through problem solving.

- 6.03 Use reasoning to solve problems.
 - a) Understand situations and communicate mathematical problem solving.
 - b) Make estimates with appropriate ranges.
 - c) Reflect, extend and refine thinking.

- 6.04 Use the language and symbols of mathematics and appropriate technology to:
 - a) solve problems;
 - b) communicate mathematical ideas;
 - c) demonstrate understanding of problems and solutions through oral, pictorial, and written explanations.

- 6.05 Create and use representations to organize, record and communicate mathematical ideas.

Grade 5

Major Concepts/Skills

- Number sense 0.001- at least to 1,000,000
- Addition and subtraction of non-negative rational numbers
- Fluency with *all* whole number operations
- Properties of geometric shapes
- Angle measurement
- Elapsed time
- Unit conversions within the *same* system
- Stem-and-leaf plots
- Simple equations and inequalities
- Number properties
- Order of operation

Concepts/Skills to Maintain

- Whole number computation
- Transformations
- Median, mode, and range
- Perimeter and area
- All graphs from previous years
- Coordinate grids

Strands: Number and Operations, Measurement, Geometry, Data Analysis and Probability, Algebra, Problem Solving and Reasoning

COMPETENCY GOAL 1:

The learner will build an understanding of and compute with non-negative rational numbers (.01 to at least 1,000,000).

Objectives

- 1.01 Develop number sense for rational numbers from 0.001 at least to 1,000,000.
- a) Demonstrate multiple ways to represent numbers using models, words and symbolic representations.
 - b) Identify the place and the value of a given digit in order to determine the magnitude of the number.
 - c) Compare and order (including the use of symbolic notation).
 - d) Identify factors and multiples including square, prime, and composite whole numbers to 100.
- 1.02. Develop fluency and flexibility with all whole number operations (including, but not limited to, standard algorithms) involving:
- a) up to three-digit by two-digit multiplication (larger numbers with calculators).
 - b) up to three-digit by two-digit division with and without remainders (larger numbers with calculators).
 - c) estimation of products and quotients and justification of the reasonableness of solutions in meaningful contexts.
 - d) analyzing the relationships among operations.

- 1.03 Develop fluency with addition and subtraction of decimals and fractions with unlike denominators (within fraction families):
- halves, fourths, eighths, sixteenths and mixed numbers;
 - thirds, sixths, twelfths and mixed numbers;
 - fifths, tenths, hundredths, thousandths and mixed numbers.
- a) Develop and analyze strategies for adding and subtracting numbers.
- b) Estimate sums and differences and justify the reasonableness of the solutions in meaningful contexts.

COMPETENCY GOAL 2:

The learner will develop fluency with standard units as they apply measurement concepts in multiple problem solving situations.

Objectives

- 2.01 Use appropriate standard units and tools to develop fluency and flexibility with unit conversions within *same* systems of measure; solve problems using these skills.
- 2.02 Identify, estimate, and measure the angles of plane shapes using appropriate tools.
- 2.03 Solve problems using the concepts and procedures involving elapsed time.

COMPETENCY GOAL 3:

The learner will understand and use properties and relationships of two- and three-dimensional shapes.

Objectives

- 3.01 Identify, describe, analyze, compare, and classify triangles and quadrilaterals by properties including sides, angles and diagonals.
- 3.02 Make and test conjectures about polygons involving:
- a) parallelism and perpendicularity of sides, and
 - b) sum of measures of interior angles.
- 3.03 Use spatial reasoning to analyze three-dimensional shapes.
- a) Describe the number of edges, faces, and vertices of polyhedra.
 - b) Relate a three-dimensional shape to its two-dimensional representation (net).
- 3.04 Explore concepts of volume and surface area for rectangular prisms.

COMPETENCY GOAL 4:**The learner will analyze data representations using statistical concepts.****Objectives**

- 4.01 Use the process of statistical investigation.
- a) Pose questions, formulate hypotheses, and design studies involve single or multiple sets of data to investigate and verify hypotheses.
 - b) Collect, organize, analyze, and display data using various representations, including stem-and-leaf plots.
 - c) Analyze data using measures of center (mode, median) and variability (minimum and maximum values, unusual data points, and range).
 - d) Explore the mean as a measure of center and its interpretation as a fair share.
- 4.02 Compare and contrast different representations of the same data, discuss the appropriateness of each representation for the context.

COMPETENCY GOAL 5:**The learner will demonstrate an understanding of patterns, relationships and elementary algebraic representations.****Objectives**

- 5.01 Analyze nonnumeric and numeric growing patterns.
- a) Use rules to describe these patterns as functional relationships (arithmetic sequences only).
 - b) Create, extend, and find missing terms.
 - c) Display numeric results using coordinate graphs.
 - d) Write equations with symbolic rules.
- 5.02 Model, write and evaluate whole number equations and equations involving addition/subtraction of decimals and fractions.
- a) Represent a problem including using variables to represent unknown quantities.
 - b) Demonstrate an understanding of equality and inequality.
 - c) Find the value of variables.
- 5.03 Develop and test generalizations based on observations of patterns and relationships:
- a) identity property for addition and multiplication,
 - b) associative property for addition and multiplication, and
 - c) distributive property.
- 5.04 Apply order of operations in meaningful contexts.

COMPETENCY GOAL 6:

The learner will make connections, solve problems and reason mathematically.

Objectives

- 6.01 Recognize and apply connections among mathematical ideas.
 - a) Connect concepts and skills from previous years to current objectives.
 - b) Connect concepts and skills from multiple strands to solve problems.

- 6.02 Develop fluency in solving single and multi-step problems that arise in mathematics and in other contexts, building mathematical knowledge through problem solving.

- 6.03 Use reasoning to solve problems.
 - a) Understand situations and communicate mathematical problem solving.
 - b) Make estimates with appropriate ranges.
 - c) Reflect, extend and refine thinking.

- 6.04 Use the language and symbols of mathematics and appropriate technology to:
 - a) solve problems;
 - b) communicate mathematical ideas;
 - c) demonstrate understanding of problems and solutions through oral, pictorial, and written explanations.

- 6.05 Create and use representations to organize, record and communicate mathematical ideas.