The learner will understand and use linear relations and functions.

5

Notes and textbook references

5.01 Develop an understanding of function.

a) Translate among verbal, tabular, graphic, and algebraic representations of functions.

A. Graph, Table, Equation (Blackline Masters V - 8 and V - 9)
The Blackline Master should be copied on an overhead transparency and cut apart. Each row has a matching table, graph, and equation.

Activity: Each group of students should be given one graph, one data table, and one equation that do not match. The challenge for the groups of students is to work together to match the correct table, graph, and equation. When the group has the correct 3 matching pieces, the students should display them on the overhead and explain to the class why they match.

Questions:
1. How is the rate of change identified from a table?

2. How is the rate of change identified from a graph?

3. How is the rate of change identified from an equation?

Notes and textbook references

B. **Spaghetti** (Blackline Master V - 3)

Materials: Graph paper, dry spaghetti or pick-up sticks, Scotch Tape™

Activity: Have the student place the graph paper on the floor beside their desk. Randomly drop the spaghetti stick on the graph paper and tape it down where it falls.

Find coordinate pairs of the spaghetti stick (points on the line) and make a table of data points. Find the linear equation to model this location algebraically using two of the points (objective 5.02). Confirm the equation using the data analysis procedure on a graphic calculator by entering several data points in the lists and finding a linear regression. Make a prediction for the y-coordinate if x was 100.

Have students compare their lines and equations.

Questions:

1. What is the rate of change (slope) in your equation? Show the rate of change on the graph by drawing several triangles (rise over run) along the line.

2. What is the y-intercept of your equation? Confirm that this is where your spaghetti crosses the y-axis.

Extension:

Turn the spaghetti graph 90 degrees counter clockwise. (This is a rotation.) The original x-axis and y-axis will now be the y-axis and x-axis, respectively. Find the equation of this line. Graph both lines on the graphics calculator. What do you conjecture about these two lines?

Teacher note: These lines would be perpendicular. The slopes should be negative reciprocals of each other. This is more clearly seen on the graphics calculator if the window is set for ZOOM SQUARE.

C. **Floor Grids (**Blackline Master V - 2)

Materials: Floor with tiles or area gridded off with masking tape

Activity: Locate the axes in the middle of the floor using masking tape. Assign 10-15 students a number such as {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}. This will represent the x-coordinates. Teacher will display one equation from the Blackline Master such as y = x and students will substitute their assigned x-coordinate value and compute their corresponding y-coordinate and then move to that location on the floor-grid. Students who are "out-of-line" will easily see their mistake.

Challenge the students by including rational numbers such as {-2.5, -1.5, -0.5, 0.333..., 0.8} If necessary, facilitate using rational numbers by changing the scale on the floor so that each tile represents 0.5 unit.

Extension:

Use a video camera and the football field to do this. Adapt the x-coordinates to yards. Students will enjoy seeing the "replay".

D. **Toothpicks** (Blackline Master V - 12 through V - 14)

Have students build the shapes shown in Toothpick Task One using toothpicks or have them draw the shapes on grid paper. As students build the shapes have them fill in the table. Students should continue building the next shape in the sequence and listing the appropriate information in the table. As students complete several shapes they should begin to recognize how they are progressing from one term to the next and make predictions on future terms in the sequence without having to build the shape. Move students to writing a "word rule" for determining the perimeter and area of any shape in this sequence. Lead the students from the word rule to an algebraic expression. Students will come up with many different representations of the same expression when simplified. This lends itself to a great discussion about what the variable represents and how each student derived their rule. Follow the same procedures for Toothpick Tasks Two and Three. The expressions for finding the areas of the shapes in Tasks Two and Three lead nicely into a discussion of nonlinear relationships.

Notes and textbook references

Notes and textbook references

b) Identify relations and functions as linear or nonlinear.

A. Ranger (Blackline Master V - 10)

Teacher should copy one of these graphs on the board and ask a student to walk in front of the motion detector (Ranger) so that the time/distance graph is similar. (Note: The circle is not a function and therefore, cannot be done in one graph, but it is fun to let students try to do it. The circle is a composite of: $y_1 = \sqrt{3 - x^2}$ and $y_2 = -\sqrt{3 - x^2}$

A Calculator Based Ranger (CBL) / motion detector (available for about $90 from distributors of Texas Instruments products **www.education.ti.com** or 1800-TI CARES)

Activity: Explicit directions for this activity come with the ranger. Connect the ranger to the graphics calculator and the graphics calculator to a viewscreen that projects the image on a screen or whiteboard. Press the APP key and access the CBL/CBR APP on the graphics calculator to run the Ranger. Place the Ranger on a desk and clear a path of about 8 feet in front of the sonic eye. The teacher should draw a graph on the board. A student should stand in front of the Ranger and will walk either away from the Ranger or toward the Ranger depending on the graph that they are attempting to copy. They will have about 6 seconds to do this. Students should explore walking at a constant pace (rate of change) and walking at a nonconstant pace. A student can use trial and error until they are able to get a correct graph.

Questions:

1. What kind of graph is displayed when you walk toward the Ranger at a constant rate of change?

2. What kind of graph is displayed when you walk away from the Ranger at a constant rate of change?

3. What kind of graph is displayed when you walk toward the Ranger at a nonconstant rate of change?

4. What kind of graph is displayed when you walk away from the Ranger at a nonconstant rate of change?

5. How does the graph change when you walk toward the ranger at a slow constant rate of change compared to walking toward the ranger at a faster constant rate of change?

Notes and textbook references

6. What would you predict the graphs would look like if you walked away from the range comparing a slow constant rate of change with a faster rate of change?

Extension:
After walking a linear graph using the ranger, trace the data, record two data points, and manually compute the linear equation (Objective 5.02). Put the equation in y = on the graphics calculator and graph it over the data from the ranger. Is the model a good fit?

B. Fahrenheit Conversion

Materials needed: graph paper, Celsius and Fahrenheit temperature scales
The relationship between the Celsius and Fahrenheit temperature scales is an example of a linear relationship. Introduce the lesson with a discussion of Fahrenheit and Celsius units of measurement. You may want to point out to students that most of the world uses Celsius as a unit of measure. Therefore, it may be useful to have the ability to convert a Celsius temperature to a Fahrenheit unit. Provide students with a thermometer that has both a Fahrenheit and Celsius scale. It is not necessary to use a real thermometer; you could draw a thermometer with the Fahrenheit temperature on one side and the corresponding Celsius temperature on the other. Make a table of equivalent Fahrenheit and Celsius temperatures. Graph the data on graph paper, treating the Celsius temperature as the independent variable (x-value) and the corresponding Fahrenheit temperature as the dependent variable (y-value). Encourage students to write a rule, or equation, that represents the relationship between the two temperature scales. Ask students to identify whether the relationship is linear or nonlinear based on their tables, graphs, and equations.

C. Going Around in Circles

Materials needed: graph paper, circles, rulers, tape measures
Ask students to determine whether these relationships are linear or nonlinear. Here are a couple of other examples of relationships that are linear. Provide students with a set of circles. Ask students to measure the diameter and calculate the circumference of the circles. Compare the diameter of circles to their circumferences. Another example of a linear relationship that students could explore is the relationship between their heights and shoe sizes. Compare the relationship of students' shoe sizes to their heights. You could also compare the relationship of students' arm spans to their heights.

Notes and textbook references

D. The Million Dollar Mission (from NCTM's *Illuminations*)

In this lesson found on NCTM's website, students are given the choice of two salary options: one cent on the first day, two cents on the second day, and double your salary every day thereafter for the thirty days or exactly $1,000,000. In this lesson, students will be able to make predictions based on patterns. This task allows students' to explore nonlinear functions. The nonlinear function in this task is exponential.
website: http://illuminations.nctm.org/swr/review.asp?SWR=3322#

E. Diagonals of a Regular Polygon

Materials needed: graph paper (Blackline Masters V - 3 theough V - 6), paper, colored pencils, markers, or crayons (optional)
It is important that students become familiar with nonlinear functions, as well as linear functions. The following is way to introduce nonlinear functions to students. Have students draw the following regular polygons: triangle, quadrilateral, pentagon, hexagon, heptagon, and octagon. You may want to provide students with these figures. Students should draw the diagonals of each regular polygon and record the number of diagonals. It may be helpful for students to use colored pencils, markers, or crayons to identify the diagonals as they count. Create a table, graph, and equation that model the relationship between the number of sides of each regular polygon and the number of its diagonals. Explore the relationship between the number of sides of each regular polygon and the number of its diagonals. This nonlinear relationship is quadratic.

c) Find, identify, and interpret the slope (rate of change) and intercepts of a linear relation.

Notes and textbook references

A. Rope Knots

Materials needed: About 6 lengths of rope, each about 3 feet long, of different thicknesses, measuring tapes, paper, pencil, graphics calculator

Activity: This activity models the decreasing amount of rope as knots are tied in the rope. Distribute a length of rope to each group of students. The student should measure the entire length of rope and record data as (0, length in either cm or inches) in a table. They will tie one knot in the rope, measure the length again, and record the data as (1, length after one knot). Tie a second knot in a different section of the rope, measure again, and record the data. Continue this process about 5-8 times to get sufficient data to find a good model (regression equation) to fit this phenomenon. Different thicknesses of rope will permit different number of knots before running out of length. Students should enter data in a graphics calculator and fit a linear regression equation.

Questions:

(1) What is the model (regression equation) and how does it compare with groups who had ropes of different thicknesses?

(2) What is the rate of change and what does it mean in this activity? How does your rate of change compare with students who had a rope of different thickness? Why is it negative? Explain.

(3) What does the y-intercept mean is this case?

(4) How much length would be left after 24 knots are tied?

Teacher note: The last question may not have an answer if the rope does not have sufficient length. Also ask students about whether or not to connect the points on the graph.

Notes and textbook references

B. Vase

Materials needed: 6-7 cylinder-shaped vases of different diameters and heights
Rulers
Container of water and measuring device that allows students to measure 25 mm of water
Paper towels
Paper, pencil, graphics calculator or graph paper.

Activity: This activity enables students to develop a model that shows the relationship of a specific amount of water in a vase to the height of the water in the vase. Distribute one vase to each group of students. Each group of students will add water to their vase in 25 mm increments and measure the distance from the bottom of the vase to the waterline. They will record the data in a table each time they repeat this procedure (5-8 repetitions). Students should then graph the resulting data (mm of water, height of waterline) and discuss the existence of a linear relationship (model) for this set of data.

Teacher note: The teacher should not initially tell the students to measure the thickness of the glass base to the vase without any water added. The students should eventually be able to define this as the y-intercept.

Possible Discussion Questions:

(1) Describe the meaning of the rate of change (slope) and the y-intercept in this situation. Do all groups have the same slope and y-intercept? Explain.
(2) Can you use the model to predict the height of the waterline after adding 25 mm of water 10 times? 20 times? (This question may need to be adjusted depending on the height of the vase.) Student should add the additional water and measure and compare the actual value with the predicted value. Discuss any discrepancies.
(3) How do graphs of vases with smaller diameters compare with graphs of vases with larger diameters? By looking at the graphs, could you tell which graph belonged to a vase with a smaller diameter? How?

Assessment:
Have each group graph their information on an overhead transparency. Place the vases in front of the classroom. Put one graph on the overhead and have students select the vase that matches the graph.

Extension:
Collect additional vases that are varied in shape, not cylinders. Do this same experiment. The resulting graphs will be nonlinear and students should discuss how the shape of the vase affects the graph.

C. Cell Phone Task (Blackline Masters V - 15 through V - 17) (adapted from *Principles and Standards for School Mathematics*. Reston, VA: NCTM, 2000)

The concepts of slope and y-intercept are often difficult for students to understand. Having students examine these concepts in a context that they are familiar with will add meaning and purpose. Allow students to work in small groups to complete this task. Students can use tables, graphs or equations to solve the problem, but should be led to writing an equation by the conclusion of the activity. Make sure to have a whole class discussion about the extensions.

Notes and textbook references

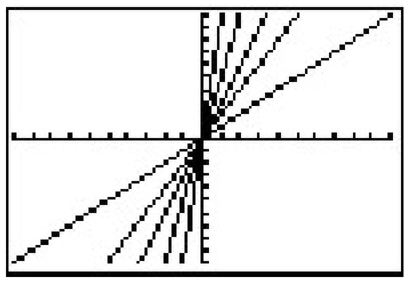
Notes and textbook references

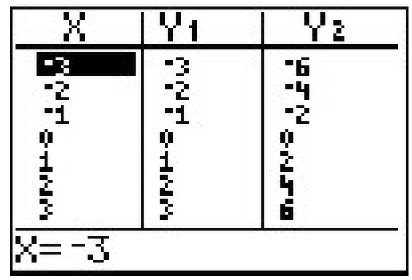
d) Interpret and compare properties of linear functions from tables, graphs, or equations.

A. Graphing Lines

Materials needed: Graphics calculator

Activity:
1. On the graphics calculator, set a standard window (ZOOM 6). In the y= menu, enter several linear equations. Graph them and discuss how changes in the slope affect the graphs.





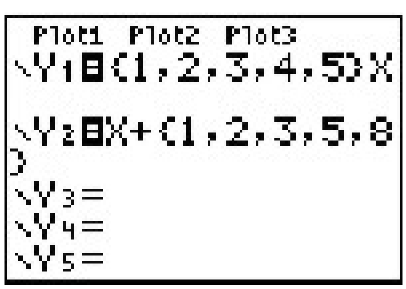
Set up a table and cursor through table values to see rate of change. Tables for the other equations can be seen by cursoring to the right.

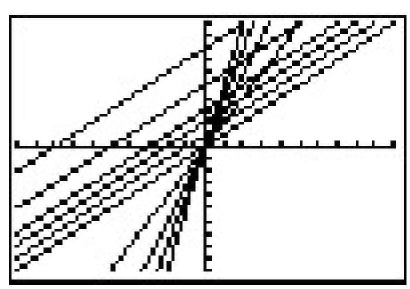
2. Explore the effect using negative slopes on a series of lines.

3. Explore the effect using fractional slopes for the series of lines.

4. Explore the effect using different y-intercepts such as y=x =1, y=x+2, etc.

5. The graphics calculator also has a list (brace) feature that allows you to graph this in one step. However, the table screen will not access all the different lines.





B. **The Picture Tells the Linear Story** (Blackline Masters V - 18 through V - 20) Students investigate slopes and y-intercepts for linear relations. This activity helps students develop the concept of parallel lines and how the "steepness" of the graph of a line relates to the slope.

C. **Moving on the Graph** (Blackline Master V - 21) Students begin with the basic equation y=x and are asked how to change the equation to meet various specifications. This allows teachers to assess what students know and understand about the concepts of slope and y-intercept.

Notes and textbook references

Suggested Classroom Accommodations for Students with Specific Learning Disabilities

Cognitive Strategies	Behavior	Accommodations
Remembering	forgets order of steps	chart of steps displayed
Self-managing	cannot explain concept	self-questioning taught
Information gathering	does not understand on first listening	frequent summaries paraphrasing strategy
Organizing	cannot make visual representation	vocabulary recorded with both words and a visualization strategy
Analyzing	cannot locate errors	verbal rehearsal strategy
Problem solving	cannot shift strategies	demonstrate each problem using two strategies
Time managing	poor assignment completion	prioritize assignments; required time chart for increased awareness of time demands
Integrating	poor notes	note taking strategy organized by concepts, not textbook chapters
Generating	weak concept connecting	prediction strategies pattern awareness
Evaluating	poor test taking	alternate tests; frequent assessment; test taking strategies

Some Additional Accommodations

- ♦ Modify original task to meet the needs of handicapped students.
- ♦ Provide taped material to listen to, rather than read.
- ♦ Emphasize higher use of objective test in contrast to subjective tests.
- ♦ Provide highlighted text for student use.
- ♦ Provide large print materials.
- ♦ Increase allowable time for completion.
- ♦ Reduce weight of test importance.
- ♦ Change fill-in-the-blank to multiple-choice format.
- ♦ Offer three choices instead of four in multiple-choice formats.

5.02 Write an equation of a linear relationship given: two points, the slope and one point on the line, or the slope and y-intercept.

Notes and textbook references

A. Equations Rope/Vase Have students do the rope activity or the vase activity as described for objective 5.01c, but use two data points to determine the model (equation). Then use the graphics calculator to determine the regression equation and compare the manually computed equation with the regression equation. Put both in the y= in the graphics calculator and graph them over the data. Compare and contrast the two equations for accuracy and for predicted values.

B. Slippery Slopes (Blackline Masters V - 4 through V - 6) Using individual dry erase coordinate planes, have students plot a pair of points as well as determine the slope of the line (formally using the equation or informally by counting the rise and run) created by the two points. Then by connecting the two points (and possibly extending the line) students should be able to estimate the value of y-intercept and therefore write an approximate equation of the line.

If you do not have a class set of coordinate planes you could make a set by enlarging a coordinate plane and laminating it with a piece of construction paper on the back. Students would then be able to use overhead or dry erase markers.

C. How Do They Fit? (Blackline Masters V - 22 and V - 23) Students use their knowledge of slope and y-intercept to write equations in standard form. Allow students to work in pairs or small groups to assemble the 3 by 3 puzzle. Teachers may want to cut the puzzle pieces apart and put them in envelopes ahead of time as the blackline is assembled correctly. Use the blank blackline to create other puzzles that involve different skills or have students create their own puzzles.

Notes and textbook references

5.03 Solve problems using linear equations and inequalities; justify symbolically and graphically.

A. **Solving Inequalities Square Puzzle** (Blackline Master V - 1) Students reassemble nine small squares into a large square. When completed, touching edges will contain an inequality and its solution.

B. **Anti-Freeze** Search for real-world problems.

Problem: Antifreeze is added to a car's cooling system to lower the freezing point to -35 degrees and raise the boiling point to 125 degrees Celsius. This can be written in an inequality: -35 < C < 125. Write this inequality in degrees Fahrenheit.

Students should recall the freezing and boiling points for each of these scales.

	FAHRENHEIT	CELSIUS
Boiling point	212	100
Freezing point	32	0

A blackline for graphing one-dimensional inequalities can be found on Blackline Master IV - 3.

Using these two data points, students should determine the linear regression equation which is the formula for conversion. This can be either manually (Objective 5.02) or using the graphics calculator. Manually students will get the equation C=(5/9)(F-32). A regression equation on the calculator will get y1= -17.7777777777778 + .55555555555556x. Students should show that these are equivalent equations.

Solution: $-35 < 5/9(F - 32) < 125$

Multiply by inverse of 5/9 $9/5[-35 < 5/9(F - 32) < 125]$

$-63 < (F - 32) < 225$

Add 32 to all parts $32 + -63 < 32 + (F - 32) < 32 + 225$

$-31 < F < 257$

Conclusion: The coolant will remain liquid between -31 degrees Fahrenheit and 257 degrees Fahrenheit.

C. **Hand** (Blackline Master V - 11)

Notes and textbook references

Problem: Research indicates that 95% of people have a palm width, p (in inches) that is a solution of the following inequality: $3.12 < p < 3.86$.This is important because this information is used when designing control panels on airplanes, keyboards for computers, gloves, and many other things.

1. Use the Blackline Master and determine whether the illustrated hand is a solution to this inequality.

2. Measure at least two students in your class and see if the measurements satisfy the inequality. Explain why or why not.

3. Write your favorite basketball player and ask him/her to outline his/her hand on a sheet of paper and sent it to you to use in your math class. Remember to get him/her to also include an autograph. Is he/she included in the 95% of people who fit this inequality?

D. **Height vs Arm Span** These data may already be collected if you did this activity in your teaching of scatterplots, if not, gather students' heights and arm spans. Each student should create a scatterplot of the data. Have students graph the line $y = x$. Discuss what this line represents. Are there any data points on the line? Discuss what it means for points to be below the line ($y < x$) as well as points above the line ($y > x$). This activity gives students a solid concept to build upon.

Notes and textbook references

E. Problem: Solve 6x + 7 = 13 numerically, algebraically, and graphically. Compare and contrast different methods. Extend methods to 6x + 7 > 13.

Activity:
Part I Numerical Solution:

(1) Student should guess an answer and substitute it into the equation to see if it maintains the inequality. Eventually they should acknowledge that this is not the most efficient method.

(2) Using a graphics calculator, students enter y1 = 6x + 7 and y2 = 13 and use the table of values to search when the solution in y1 equals to the solution in y2.

Part II Algebraic Solution:

	6x + 7 = 13
Add (-7) to both sides	6x + 7 + (-7) = 13 + (-7)
Inverse Prop/Number fact	6x + 0 = 6
Zero Prop	6x = 6
Mult (1/6)	6 * (1/6) x = 6 * (1/6)
Inverse Prop	1 x = 1
Identity	x = 1

Check is left for you.

Part III Graphical Solution:

Using a graphics calculator, students enter y1 = 6x + 7 and y2 = 13. Press WINDOW and set a window that allows you to see BOTH lines. Press TRACE and cursor (right or left) to the point of intersection. Press CALC (2^{nd} TRACE), select #5 Intersect. Calculator will indicate in the top left which equation it is on, so press ENTER to indicate that this is one of the lines for which you want the intersection. Notice the top left will change to the second equation; press ENTER to indicate that this is the correct second equation. Press ENTER one more time to indicate your guess with the cursor blinking close to where the lines intersect. What is the answer for x?

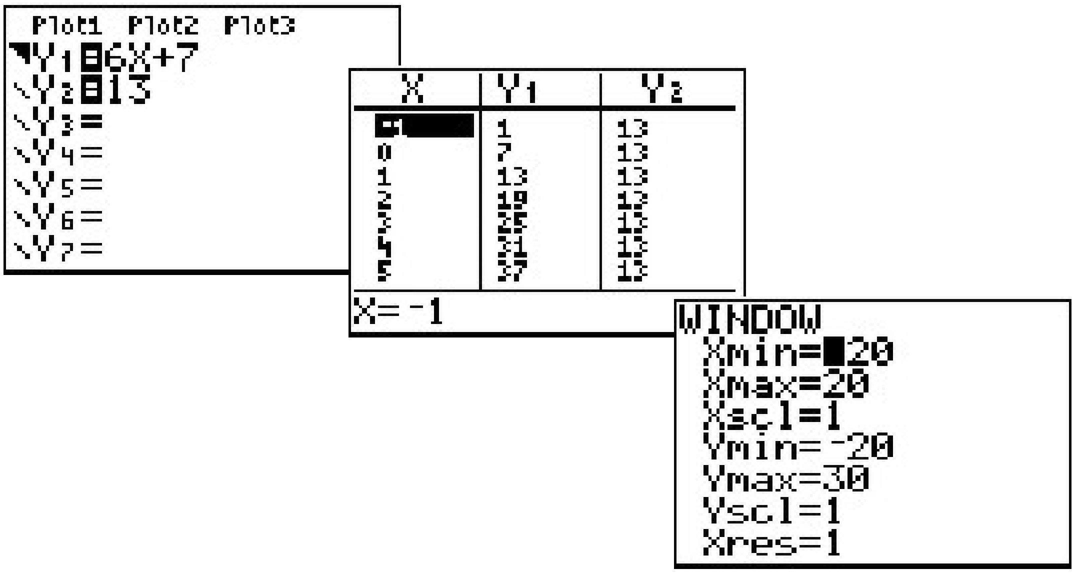
Extended problem: Find at least 6 solutions for 6x + 7 > 13.

Part I Numerical Solution and

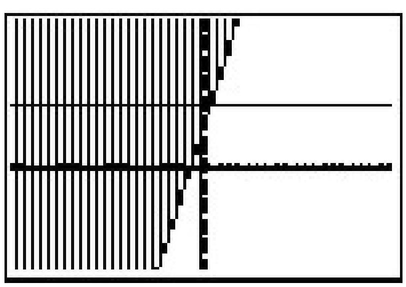
Part II Algebraic Solution

Follow the steps in the original problem to find solutions for this inequality.

Notes and textbook references



In the first screen shot, notice the icon on the left side of y1. Cursor to this and press ENTER to toggle through the possible icons. Recall the problem is 6x+7>13. Therefore, y1 will be indicated as greater than (or above the line). In the second screen shot, answers in the y1 column that are greater than 13 are solutions such as when x = 5. Substitute this in the original inequality to verify manually that you get a true inequality.
In the third screen shot, an appropriate window is set to see the graph.



Any point in the shaded area (to the left of the sloped line) and that is also above the horizontal line will be a solution to this equality. It should be reinforced that inequalities have multiple answers, both integer values and fraction/decimal values.

Notes and textbook references

5.04 Solve equations using the inverse relationships of addition and subtraction, multiplication and division, squares and square roots, and cubes and cube roots.

A. Inverse Relationships Students should have a strong understanding of inverse relationships and that they "undo" the other operation. Have them explore beginning with a random number like 8, then adding 2, then subtracting 2. Explore also with the other inverse relationships:

- Begin with 8: Add 5, then subtract 5, what is the result? Ask them to explain why this works. (They should acknowledge that a positive 5 and a negative 5 will equal zero.) Teachers should also explore Adding 5 and adding a (-5).
- Begin with 12: Multiply by 2, then divide by 2, what is the result?
- Begin with 36: Take the square root, then square the result, what do you get? Note the order of operations is changed in this problem. Students need to know that it is the combination, not the order that undoes.
- Begin with 4.2: Use the graphics calculator to take the cube root (located in the MATH menu) and cube the result.

Relate this concept to equations:
$x + 1 = 5$. Explain to solve for x, the positive 1 on the left side can be taken away by subtracting 1. However, to maintain the equivalence, you will also need to subtract 1 from the right side. It is VERY important for students to show work, detail properties / operations beside each step, and to check the answers to show the equivalence.

Problem:	$x + 1 = 5$	
Add -1 to both sides	$x + 1 + (-1) = 5 + (-1)$	
Inverse Prop/ Number fact	$x + 0 = 4$	
Add 0 Prop	$x = 4$	
Check	$x + 1 = 5$	Copy original equation
Substitute answer	$(4) + 1 = 5$	
Number fact	$5 = 5$	The equivalence has been maintained.

PROBLEMS:
- Solve $x - 4 = 9$
- Solve $2x + 3 = 4$
- Solve $0.5x - 6 = 20$

EXTENDED PROBLEMS FOR EXTENDED UNDERSTANDING:
Building on these understanding, help them work through more difficult equations.

Problem:	$x^2 + 4 = 20$	
Add -4 to both sides	$x^2 + 4 + (-4) = 20 + (-4)$	
Inverse Prop/Number fact	$x^2 + 0 = 16$	
Add 0 Prop	$x^2 = 16$	
Take Square root of both sides	$\sqrt{x^2} = \sqrt{16}$	
Inverse Prop	$x = 4$ and also $x = -4$	
Check (+4)	$x^2 + 4 = 20$	Copy original equation
Substitute	$(4)^2 + 4 = 20$	
Number fact	$16 + 4 = 20$	
Number fact	$20 = 20$	The equivalence has been maintained
Check (-4)	$x^2 + 4 = 20$	Copy original equation
Substitute	$(-4)^2 + 4 = 20$	
Number fact	$16 + 4 = 20$	
Number fact	$20 = 20$	The equivalence has been maintained.

Try these:

a) $\frac{x^2}{2} = 8$ b) $x^3 = 27$ c) $\sqrt[3]{x} = 2$ d) $2x^2 + 3 = 21$

B. **Cut & Paste Equations** (Blackline Master V - 7)

Materials Needed: Blackline Master, a sheet of construction paper, scissors, and a glue stick.
This is a great way for students to practice solving equations using inverse operations. The cutting and pasting is great for kinesthetic learners. Students should be instructed to leave the Equation column intact and then cut and paste the appropriate steps for each equation. This idea can be adapted for a variety of concepts including: one and two-step equations, inequalities, and graphing linear equations.

Notes and textbook references

Notes and textbook references

C. Scales and Equality

Materials needed: scale,weights or other objects to illustrate equality.
To introduce the concept of using inverse operations to solve equations, use a scale to model the relationship of equality. Start with a relationship that models equality by putting equal weights or objects on each side of a scale. Discuss the meaning of equality. Illustrate inequality by taking an object, or objects, from one side of the scale. What needs to be done to the other side to make the sides balance out? Do several examples. Emphasize that to regain balance you must do the same thing to the opposite side of the scale.

Extension using Algebra Tiles™

Materials needed: algebra tiles
Draw a scale. Using algebra tiles, represent one side of the equation with algebra tiles on one side of the scale, and represent the other side of the equation with algebra tiles on the other side of the scale. Use the manipulatives to solve the equation. Repeat with other types of equations.