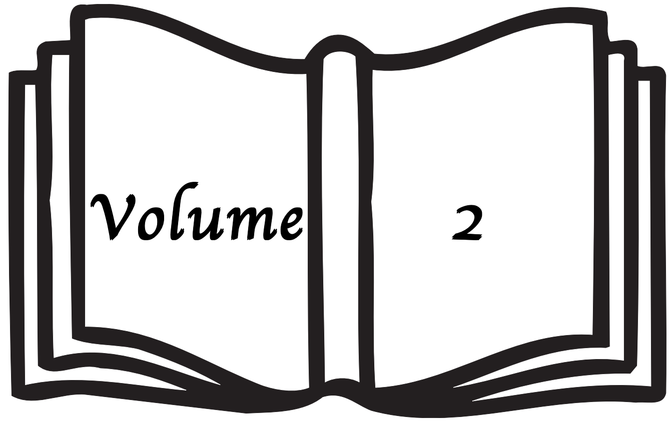
Grade Seven

Classroom

Strategies



The learner will understand and compute with rational numbers.

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1.01 Develop and use ratios, proportions, and percents to solve problems.

Notes and textbook references

A. Gulliver's Travels (Blackline Masters I - 26 and I - 27)
Gulliver's Travels by Jonathan Swift is a great way to integrate both reading and math. Use the blacklines for the excerpt, or collaborate with the Language Arts teacher and have the students read the whole book. This activity can be used as a short example of proportions and ratios in the body, or can be extended to be more comprehensive.
Note: Make sure you do this activity before the students do to ensure you can answer all the questions and are prepared for the results.

B. Asian Passport Spinners (Blackline Master I - 24)
Students use their proportional reasoning skills to create spinners based upon the likelihood of collecting nine passport stamps from different countries in Asia. The students will use proportions to find the correct angle measure or percentage for each situation described on the blackline.

C. Food Labels The websites listed below provide nutritional information from four fast-food restaurants. Students may use these fact sheets to create some interesting ratios such as calories to ounce serving, fat grams to calories, carbohydrates to calories, fat to protein, saturated fat to total fat. In the case of Wendy's and Burger King, students can look at the sandwich components and discover what happens when they make choices such as replacing mayonnaise with ketchup.

www.bk.com/food/nutrition/index.aspx

www.wendys.com/food/index.jsp

www.mcdonalds.com/usa/eat.html

www.tacobell.com

Notes and textbook references

D. **Cooperative Problem-Solving Cards** (Blackline Masters I - 3 through I - 7)Cooperative Problem-solving cards offer an opportunity for students in a cooperative group setting to have equal input into a group solution. No single member of the group can solve the problem without the information on the other group members' cards. Through discussion of the problem within the group, students can observe the thought processes of their peers. These cards are a good scaffolding tool to help students continue to participate in the problem-solving process while they grow in their ability to solve problems independently.

Students work in groups of two or three to solve problems that involve proportional reasoning. A different part of the problem is presented on each of the cards. Each student gets at least one card and reads, but does not show, that part of the problem to the group members. The group uses all of the information from the cards to solve the problem.

E. **Thousand Mile Race** (Blacklines I-8 through I-14)
The purpose of this game is to allow students to practice their computational strategies of percent problems. Students should use mental math or paper/pencil to solve the problems. This game can be played with a whole class divided into three teams or with small groups of students divided into three teams.

Materials Needed: Playing mat and playing cards which have been cut apart and placed in a paper bag (these items need to be on transparencies if playing as a whole class).

Object of the Game: To be the first team to reach exactly 1000 miles.

The Cards

Mile Cards – percent problems that are worth 50, 100, 150 or 200

Go Cards – allows a team to begin playing again if it has been in a stop or chase situation.

Stop Cards – stops an opposing team from having a turn until the team plays a Go card from its stockpile.

Chase Cards – if a chase card is played against a team that team may only answer 150 or 200 mile questions unless the team plays a Go card from its stockpile.

Notes and textbook references

How to Play

1. Draw three cards from the bag and place them face up on the playing mat in the top grid. These are the cards in play.
2. Team 1 selects one of the three cards in play.
   If they choose a **Mile Card**, they must answer the problem correctly to receive the number of miles.
   If the team chooses a **Go Card** they can stockpile it to use later.
   If the team chooses a **Chase** or **Stop Card** they may play it against an opposing team by placing it in that team's Hazard Pile.
3. Team 2 (and proceeding turns) draws a card from the bag to replace the missing card so that there are always three cards in play.
   If the team has no hazard cards against them they follow the same guidelines as outlined in step 2. If the team has a **Chase Card** against them they must answer a 150 or 200 mile question or play a **Go Card** from their stockpile and select any of the three cards in play.
   If the team has a **Stop Card** against them they can only use a **Go Card** from their stockpile and select any of the three cards in play.
   If the team has no **Go Cards** in the stockpile, the team loses its turn.
4. If a team cannot use any of the three cards in play the team loses its turn.

F. **Rational Review Triangle Puzzle** (Blackline Master I - 2)

A strategy for having students build their capacity to understand and work with rational numbers is to have discussions with their peers about the thought processes involved in mentally solving rational number problems. In this activity, students work in pairs to solve rational number problems and assemble the small triangles into the indicated shape. Each rational number problem matches exactly one answer. The problems include arithmetic operations involving rational numbers and can all be solved using mental math.

G. **Four's A Winner** (Blackline Masters I - 15 and I - 28)

Materials: Gameboard, two paper clips, two different colored sets of markers

Number of players: Two players or two teams

Directions: Player one places one paper clip on a percent expression, the second paper clip on a number, and a marker on the correct answer to the percent change of the number. The second player moves one paper clip only and places a marker on the corresponding correct answer. Play continues in this manner. The winner of the game is the first player to get four in a row vertically, horizontally, or diagonally.

Note: The gameboard could be put on a transparency and this could be used by two teams of students.

Notes and textbook references

H. Pictorially Proportional (Blackline Master I - 41)
This activity provides four proportion problems which may be better understood through the use of the diagrams provided. Students may work in pairs to solve these problems, or the teacher may wish to use them as transparencies for a group discussion of solving proportions. In order for this task to be successful, students must be the center of the task and teachers must refrain from showing students how to solve these problems. Begin class discussion by asking groups of students how they solved each problem. Have the students demonstrate the various ways that each problem can be solved.

Visual skills are useful and even necessary in understanding many areas of mathematics. It is amazing how many students do not rely on visualization and even more shocking to find how many of them are not able to illustrate math concepts with a diagram.

The diagrams provided in this exercise can help students understand proportions in several ways. They may choose to focus on the unit concept. That is, if six pairs of jeans cost $240, then how much does one pair cost. Once they solve that problem, it is easy to answer how much 20 pairs or 120 pairs will cost. Or they may look at things another way. For example, if five waiters are needed for one row of 16 tables, how many are needed for 48 tables. Three rows would be needed for 48 tables. Each row needs five waiters.

I. Flags This activity incorporates the different cultures present in your classroom. Have students bring in pictures of the flag of their native country (or have students pick a flag of another country). Using the picture, students should estimate the percentages of the different colors used. Students can further apply their skills by drawing their flag proportionally and then coloring it appropriately. The completed assignment makes a great bulletin board display.

J. Percents in the News Collect five or six articles from the newspaper that use percentages. Attach each article inside a file folder and have students write problems related to their articles. Place those in the folder. In cooperative groups, have the students work the file folder problems.

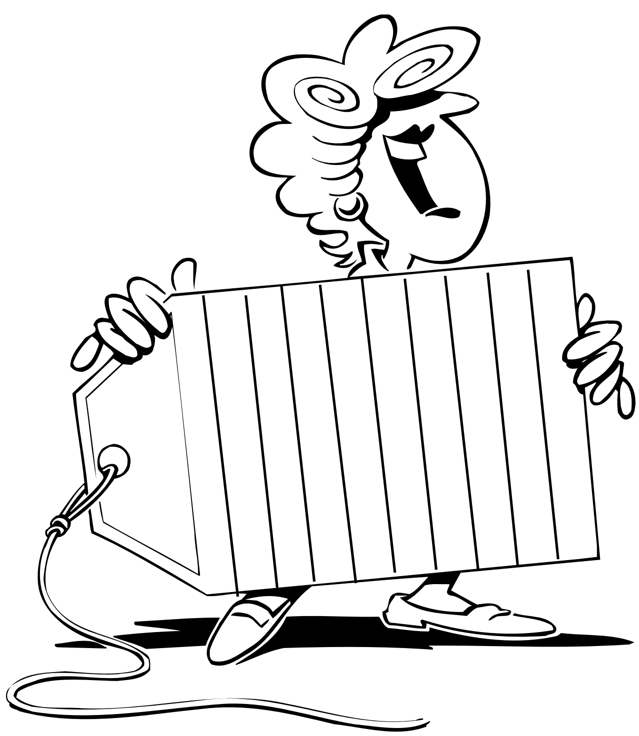
K. **Percent Models** (Blackline Master I - 33)

Notes and textbook references

While many students are successful in learning algorithms for solving percent problems, they have very little conceptual understanding of percent. They often have difficulty estimating percents, a skill used in everyday life when calculating taxes, tips, sales, etc. Percent bars are a great way for students to visualize percents and to begin to make sense of them. Percent bars can be used to estimate and to solve percent problems, including percent of change.

Introduce percent bars with the blacklines. Talk students through this by finding various benchmark percents. For example, 50% of 80 is 40 because half of 80 is 40. Student can then reason that any value less than 40 is less than 50%. Though this seems simple, students usually have not reasoned this way before. 25% of 80 is 20. Since 25% is half of 50%; 25% of 80 must be half of 40 which is 20. Other benchmarks are 75% and 10%. From these benchmarks, find other percents (20%, 5%, 60%). If a student knows 10% of 80 is 8, then 20% is 16. Similarly if 20% of 80 is 16 then 60% is three times that or 48. Student can find 5% by dividing 10% in half to get 4. Once students have done a few of these as a class, let them explore percents on their own by completing the sheet.

When students become comfortable with percent bars, they can choose to use them as a tool to make sense of percent problems and solve them effectively.



Notes and textbook references

1.02 Develop fluency in addition, subtraction, multiplication, and division of rational numbers.

a) Analyze computational strategies.

A. Integer Addition Race (Blackline Master I - 1)
Materials needed: Pawns for the players, 2 dice of different colors, playing mat. Before starting the game, decide which color die will represent positives and which will represent negatives. On a turn, each player rolls the two dice and determines the sum of the roll. The player with the largest distance from 0 (i.e. larger absolute value) moves forward one. If a winning player rolls a sum of 5 or –5, he/she will move ahead two instead of one. If the players tie, they each roll again. The first player to reach the finish is the winner.

While students are learning about fractions they surely stumble across the fact that dividing by a number is the same as multiplying by its inverse. Dividing a number by 2 is the same as multiplying it by one-half. Since reciprocals always multiply to equal one, they must have the same sign. Once we know the rules for multiplying signed numbers, the rules for dividing follow.

B. Integer Computation Square Puzzle (Blackline Master I - 16) Students reassemble 16 small squares to make a larger square. Where edges touch, an integer computation problem will match its solution. Students should work in groups of two or three for this activity. This will allow student discussion of strategies and reveal common mistakes. Teachers may want to cut the puzzle into pieces and put them in an envelope as the Blackline Master provides answers.

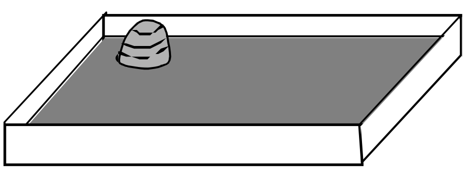
For follow-up, have students make their own square puzzle. Can they make one that will stump their peers? Students should be cautioned not to use the same answer more than one time as this makes the puzzle very difficult to solve.

C. Integer Computation Triangle Puzzle (Blackline Master I - 17) Students reassemble small triangles to make the shape shown on the blackline master. Where edges touch, an integer problem will match its solution.

D. Rational Math Bingo (Blackline Masters I - 18 through I -22 and I - 30) Each student is asked to make a bingo card with numbers as specified on the card. Each column must contain different numbers from the indicated range, but the numbers can be placed in the column in any order. It is recommended that students work in pairs to discuss and check with each other.

When the game is played, the teacher will put questions on the overhead projector. Students work the problems mentally and look for the answer on their cards. The first student (or pair) to complete a line is the winner.

E. Modeling Signed Numbers with Heaps and Holes This activity is based on a few lines from the movie, ***Stand and Deliver***. In the movie Jaime Escalante is trying to get his students to understand how negative numbers work by filling in holes in the sand. Explain to your students that +1 is like a pile (or heap) of sand on a level beach. A hole of equivalent size dug into the beach represents -1. This model explains positives as a surplus and negatives as a deficit.



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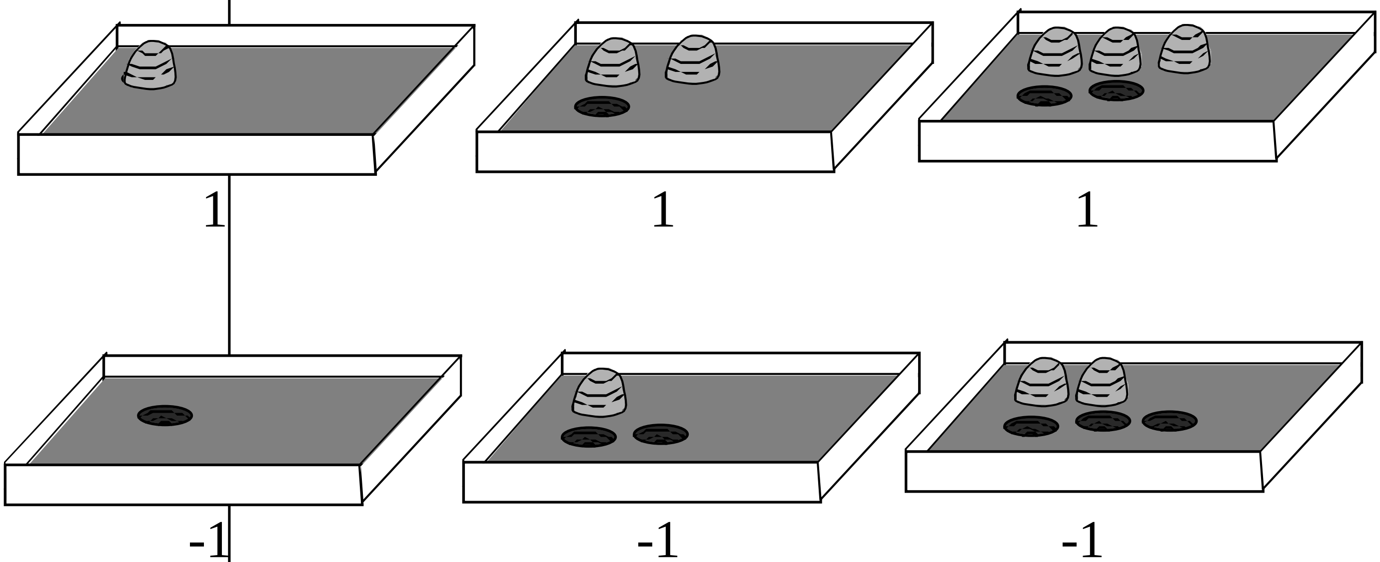
-1

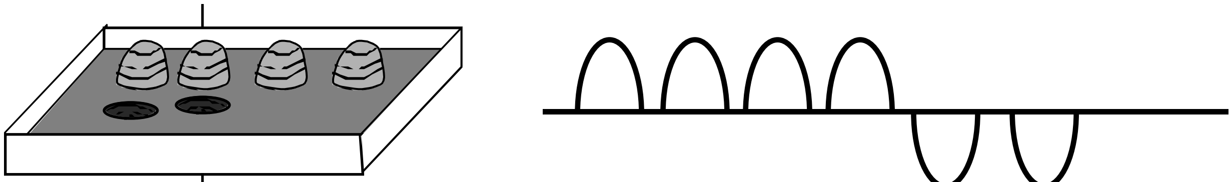
First, convince your students that there are many ways to model zero. Two are shown here.

Notes and textbook references

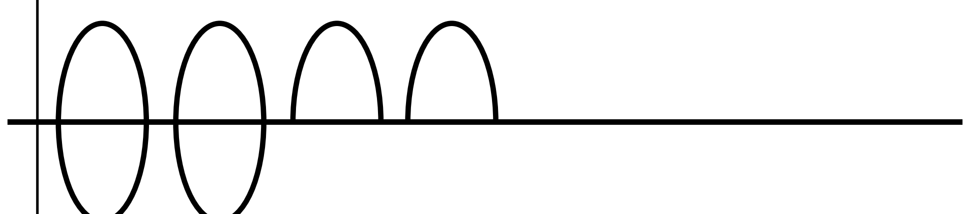
Also show them various ways to model other numbers such as (+1) and (-1).



The following illustration shows the addition of 4 + (-2). To simplify the drawing, a flat line can represent zero. Half-ovals above the line represent positive numbers (heaps) and half-ovals below the line represent negatives (holes). Here is the line drawing for 4 + (-2).



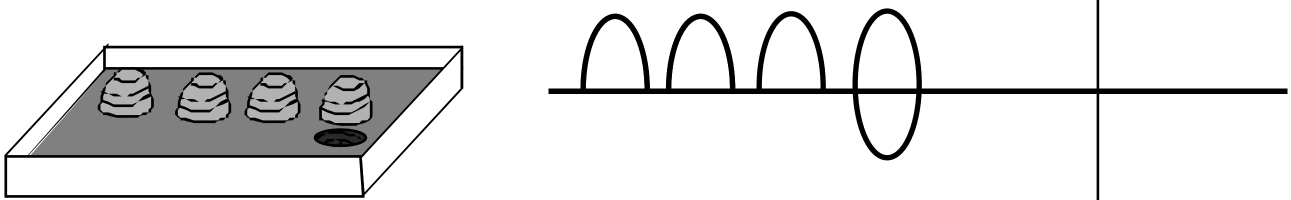
When the bottom and top ovals are lined up, a positive and a negative form something that looks like zero, and the result is displayed more clearly.

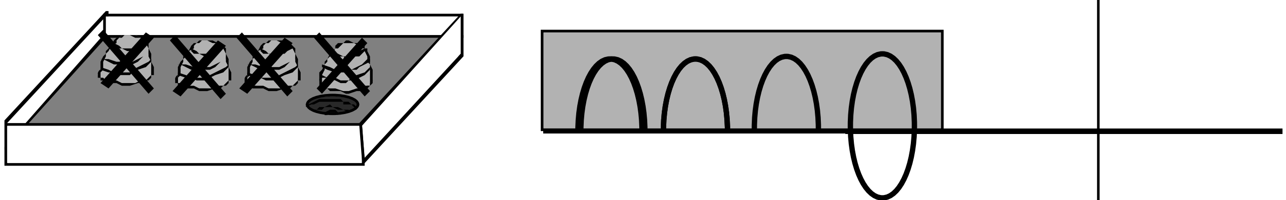


4 + (-2) = 2

There aren't enough heaps to take away 4, so we remedy the situation by adding a zero. The total hasn't changed since we added a (+1) and a (-1), but now we can take away four.

Notes and textbook references

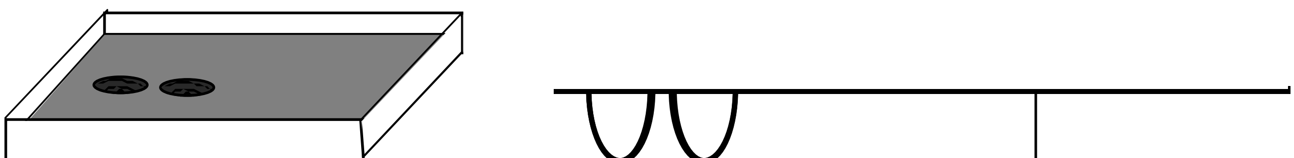




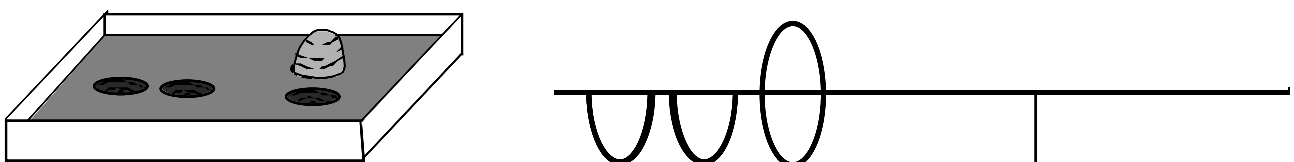
3 – 4 = -1

More subtraction examples:

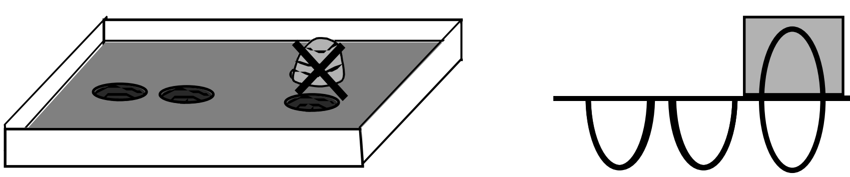
-2 -1 **Start with -2.**



There is no positive one to subtract, so we add a Heap and a Hole pair (a zero).



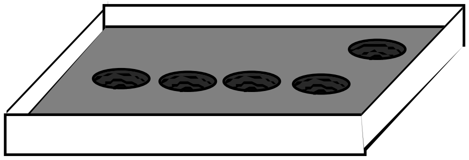
Now subtract 1.

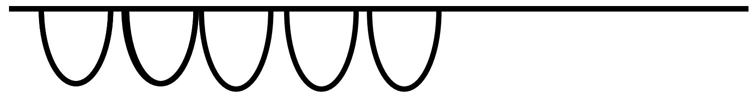


-2 –1 = -3

Notes and textbook references

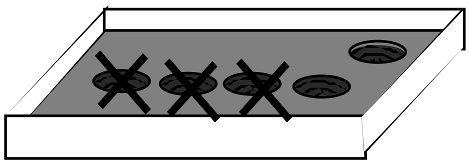
-5 - (-3) is read “Negative 5 subtract negative 3.”

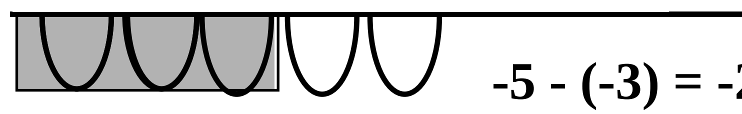




-5 **-5**

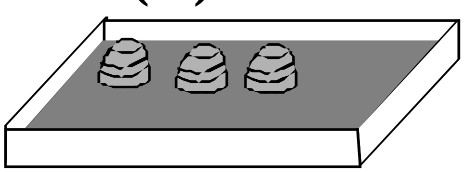
We have enough to take away (–3).



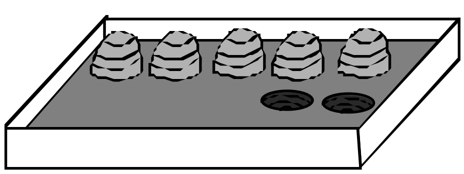


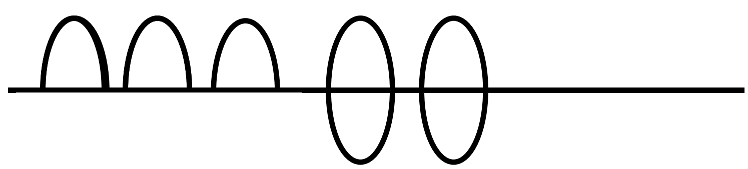
-5 - (-3) = -2

3 - (-2) **Start with 3**

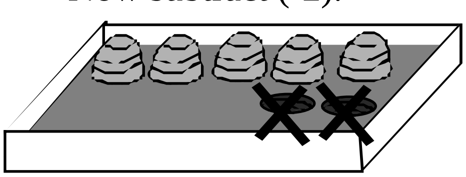


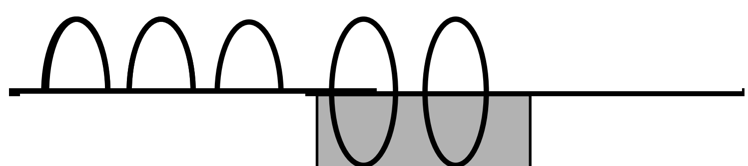
There is no (-2) to subtract so we add 2 heaps and 2 holes (2 zeros).





Now subtract (-2).





3 - (-2) = 5

Multiplying with heaps and holes line notation is easier if you think of multiplying as repeated addition if the first factor is positive or repeated subtraction if the first factor is negative.

3 x (-2) means add in 3 sets of negative 2.

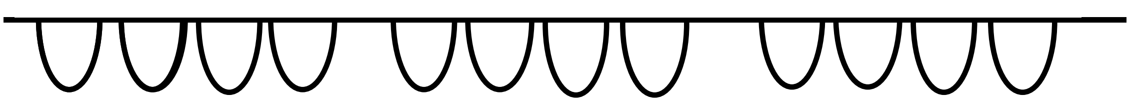
-2 x 3 means take away two sets of 3.

-2 x (-3) means take away two sets of negative 3.

Examples:

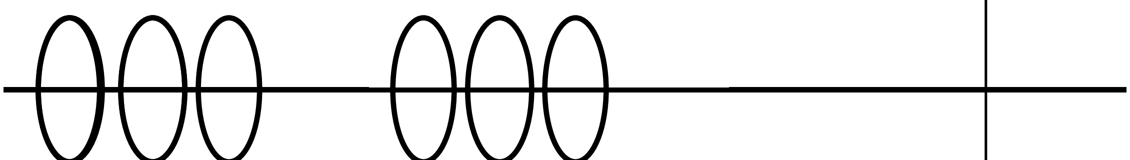
Notes and textbook references

3 x (-4) means add in 3 sets of (-4).

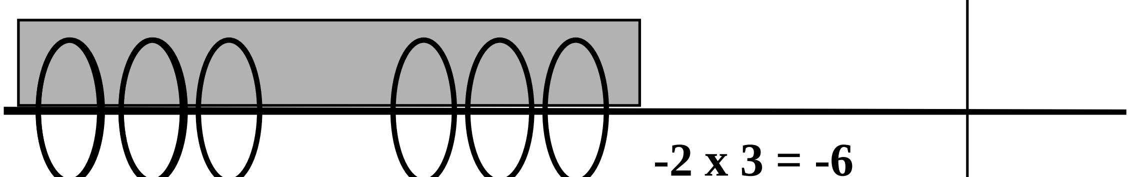


3 x (-4) = -12

-2 x 3 means take away 2 sets of 3. If we start with 0, there is no way to take away anything. But we can add additional symbols that still represent 0. Zero may be added to any number without changing the total.

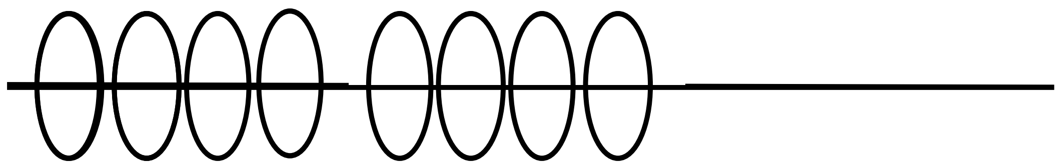


Now we can take away 2 sets of 3.

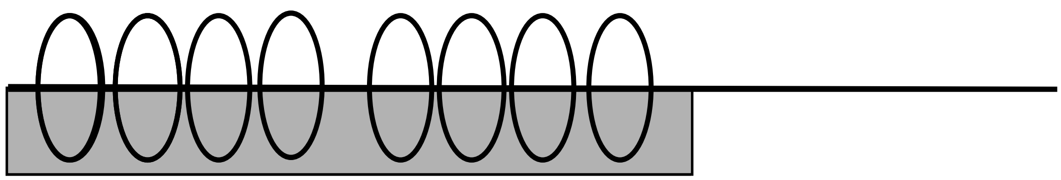


-2 x 3 = -6

-2 x (-4) means take away 2 sets of (-4). If we start with 0, there is no way to take away anything. But we can add additional symbols that still represent 0. Zero may be added to any number without changing the total.



Now we can take away 2 sets of (-4).

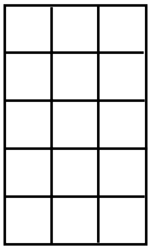


-2 x (-4) = 8

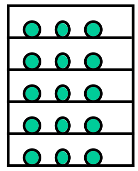
F. Modeling Addition of Fractions Students who have trouble remembering the algorithm for adding fractions can be shown how to do that with diagrams. To use this technique, students should realize that to understand a fraction, one must know what the "whole" is. Is it one pizza, one rectangle, one circle, or one candy bar? Also, the students should know how to simplify fractions.

Example: $\frac{3}{5} + \frac{1}{3}$

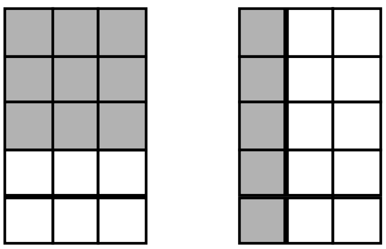
These fractions can be added easily if we have a convenient diagram of the whole. If graph paper is available, use a rectangle that is 3 x 5. If no graph paper is available, make a dot matrix that is 3 dots wide and 5 dots long. Either of these can easily be divided into thirds or fifths. This model will represent the "whole" or "one." Point out that in each model, one cell or dot is equal to $\frac{1}{15}$.



$\frac{3}{3}$

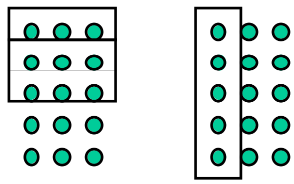


$\frac{5}{5}$



$\frac{3}{5} + \frac{1}{3}$

$\frac{9}{15} + \frac{5}{15} = \frac{14}{15}$



$\frac{3}{5} + \frac{1}{3}$

$\frac{9}{15} + \frac{5}{15} = \frac{14}{15}$

Notes and textbook references

G. **Four in a Row** (Blackline Master I - 23)

The class is divided into two teams. To start play, the teacher puts an algebraic expression on the overhead. On a team's turn, they will give coordinates for a point they wish to capture. That point is circled. If the team can then give the correct answer for substituting the coordinates into the expression, the team captures that point, and the circle is filled in with the team's color. If the team gives an incorrect solution, the opposing team gets to try to fill it in. Teams alternate playing until one team has captured four points in a row either horizontally, diagonally, or vertically.

If the leader wishes to direct students toward negative numbers, he/she may circle a point in the 2nd, 3rd, or 4th quadrant that may be used by either team as a free spot. You may wish to play several rounds with your students to determine a winner.

Notes and textbook references

H. Using Patterns to Develop the Rules for Multiplying and Dividing Integers (Blackline Master I - 29)

Begin the discussion with a series written vertically on the board such as the original pattern:

Original Pattern	***Explanation***	***Extended Pattern***	***Explanation***
4 x 5 = 20			
4 x 4 = 16	20 – 4 = 16	4 x -1 = -4	0 – 4 = -4
4 x 3 = 12	16 – 4 = 12	4 x -2 = -8	-4 – 4 = -8
4 x 2 = 8	12 – 4 = 8	4 x -3 = -12	-8 – 4 = -12
4 x 1 = 4	8 – 4 = 4	4 x -4 = -16	-12 – 4 = -16
4 x 0 = 0	4 – 4 = 0	4 x -5 = -20	-16 – 4 = -20

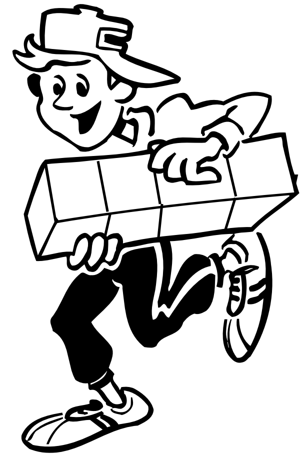
Discuss what is happening to the product in the original pattern each time the factor is decreased by 1. The product is 4 less than the previous product. This indicates that you are subtracting 4 each time. Write the explanation next to the original pattern. Extend the pattern to 4 x -1. Students should continue the pattern of subtracting 4 to get the product of -4. Continue the pattern on the board a few more times. Investigate with other factors to show students that the pattern always works.

Once the pattern that a positive multiplied by a negative results in a negative has been established, move to investigating a pattern for a negative multiplied by a negative results in a positive product. Start with a series such as:

Original Pattern	***Explanation***	***Extended Pattern***	***Explanation***
-6 x 5 = -30			
-6 x 4 = -24	-30 + 6 = -24	-6 x -1 = 6	0 + 6 = 6
-6 x 3 = -18	-24 + 6 = -18	-6 x -2 = 12	6 + 6 = 12
-6 x 2 = -12	-18 + 6 = -12	-6 x -3 = 18	12 + 6 = 18
-6 x 1 = -6	-12 + 6 = -6	-6 x -4 = 24	18 + 6 = 24
-6 x 0 = 0	-6 + 6 = 0	-6 x -5 = 30	24 + 6 = 30

The pattern this time is adding 6 to the previous product. Write the explanation next to the original pattern. Continue the pattern to -6 x -1 = 6. Write the explanation next to the extended pattern. Continue the pattern several times to show that a negative multiplied by a negative results in a positive product. Use Blackline Master I - 29 as a follow-up to the discussion.

Notes and textbook references



I. **Building Rectangles from Cubes** (Blackline Master I - 32)

Materials: 8-12 color cubes of each color (green, blue, red, yellow) for each group. If you do not have color cubes in your classroom, students may use grid paper and colored pencils to draw the rectangles.
Students should work in groups of two or three.

These tasks help students with their understanding of fractions such as the concept that the same fraction can have different names and the necessity for a common denominator when adding fractions.

J. **What Are The Rules?** Rules and facts are easily and quickly stated, but not so easily remembered and applied.

It is important that students learn the underlying mathematics through exploration with number lines, two-colored counters, or other manipulatives. Students can then determine the algorithms for operating with integers for themselves. This is much more meaningful and bears longer lasting results.

There are few concrete examples that demonstrate the rules for multiplying and dividing integers. An excellent way to "talk students through this" is using opposites. A negative sign indicates the opposite of a number. For example, -2 x 3 = -6 because 2 x 3 is 6 and the opposite (-2) of a positive (3) is a negative. Another example. -3 x -5 = 15 because
3 x 5 =15 and the opposite (-3) of a negative (-5) is a positive.

While students are learning about fractions, they will surely stumble across the fact that dividing by a number is the same as multiplying by the multiplicative inverse (commonly called the reciprocal). Dividing a number by 2 is the same as multiplying by one-half. Thus, the integers rules for multiplication also work for division.

K. **Four in a Row** (Blackline Master I - 38)

Students will need game boards, markers of two different shapes or colors, and two paper clips. Play begins by the first player placing the two paper clips on any pair of factors along the bottom edge of the game board. The player then places a marker on the square which is the product of the two factors. The next player is allowed to move exactly ONE clip and cover the square which is the product of the two indicated factors. (Both clips can be placed on the same factor to square that factor.) Play alternates until some-one gets four markers in a row, horizontally, vertically, or diagonally. The teacher may want to demonstrate the game on the overhead with the class before students play one another.

Notes and textbook references

L. **Lining Up Dominoes** (Blackline Masters I - 39 and I - 40)
Students will make a train of dominoes by matching a problem to its solution. It may be helpful to cut the dominoes apart and put them in envelopes ahead of time as the blackline is assembled correctly. The blank domino sheet can be used for students to make their own dominoes. You may have them include other operations or concepts.

M. **I Have, Who Has** (Blackline Masters I - 42 and I - 43)
Students will listen, perform operations, and respond when appropriate in a round-robin format. You may allow students to use paper and pencils to do the computations with integers or you may use this as a mental math activity.

b) Describe the effect of operations on size.

Notes and textbook references

A. Before beginning this activity, ensure that students are familiar with these vocabulary words: addend, sum, minuend, subtrahend, difference, factor, product, divisor, dividend, and quotient.

List the following statements on the board. Students are to fill in the blanks with always, never, or sometimes.
a. When I add two numbers, the sum is _______________ larger than the addends.
b. When I subtract two numbers, the difference is ____________ smaller than the minuend.
c. When I multiply two numbers, the product is _________________ larger than the factors.
d. When I divide two numbers, the quotient is _________________ smaller than that dividend.

In all cases, the answer is sometimes. When adding two positives, the sum is always larger than the addends, but if I add a positive and a negative, the sum is less than at least one of the addends. Similarly, subtracting negatives increases the difference. In multiplying and dividing, arguments can be made with integers, but they can also be made with rational numbers. Multiplying by a fraction less than one will result in a smaller product than factors. Conversely, dividing by a fraction less than one will result in a larger quotient.

From students' answers, begin a class discussion. If a student says always, challenge others to find a counterexample. If a student says never, challenge others to find a possible solution. If a student says sometimes, challenge them to determine for what conditions this is true and when it is not. This can lead to rich mathematical discourse about the affect of operations on size.

B. (Blackline Masters I - 34 and I - 35)
Before beginning this activity, ensure that your students have a working knowledge of the following vocabulary words: addend, sum, minuend, subtrahend, difference, factor, product, divisor, dividend, and quotient. Have students work in groups to complete the blacklines. Afterwards, lead a class discussion of the results that students found.

Notes and textbook references

High school teachers often complain that students don't have sufficient math sense. They are often equating math sense with a student's ability to work rational number problems without a calculator. In adult life students will usually have access to a calculator, but will often rely on mental computation and estimation. Math sense, confidence and competence can be supported through encouraging students to use mental math.

C. Students often have the misconception that a product is always larger than its factors and a quotient is always smaller than its dividend. One way for students to investigate this concept is through the use of models. After exploration with models students can begin to make generalizations about the types of factors that produce products that are larger than its factors and products that are smaller than its factors. Generalizations can be made about quotients also.

An example of an exploration that could be done includes the use of grid paper. Begin with multiplying a whole number by a decimal such as 3 x 0.16. On a 10-by-10 grid have students shade in 16 hundredths a total of three times. They will see that only 48 of the 100 boxes are shaded. Have students model several other multiplication problems in a similar way. Lead a discussion about why the product is always less than the whole number. Students should be able to reason that this happens because they are multiplying by a number that is not even one whole. Remind them that any number multiplied by one is the number. This should help them understand that multiplying by a number that is less than one will produce an answer that is less than the whole number they started with.

A similar investigation can be done with division of fractions. Use a problem such as six divided by two-thirds. Draw six boxes each with three equal sections. Have students shade in two sections at a time as many times as possible. You may have them use different colors each time so they can easily see how many two-thirds sections they made. Students should have nine two-thirds sections shaded. Try this method with other whole numbers divided by fractions. In a discussion about why the quotient is larger than the dividend remind students that dividing by one whole produces the same number as the dividend. When dividing by a number smaller than one whole you will get more groups than what you started with.

Notes and textbook references

Tips for Problem Solving in Your Class

- Set the expectation that everyone thinks! State a problem and then give everyone a moment to think about it.
- Use think-pair-share to jumpstart your students' problem-solving processes. First they think over the question, then they talk it over in pairs, then each pair shares with a larger group.
- Don't let textbooks or other published supplementary materials thwart the problem-solving process. Be wary of texts that give many drill problems with one word problem that is solved the same way as the previous problems. Also watch out for problem sets that are all basically identical.
- Incorporate group problem solving into your lessons, so students have a chance to observe their peers.
- Use problems from a variety of sources. Ask questions in a variety of ways.
- Ask a variety of questions from the same problem source data. Students begin to anticipate what a question will be without having really read the problem. Keep them flexible in their expectations.
- Expose students to problems in which the numbers they read in the problem are not necessarily the ones they will "crunch" to solve the problem. Use price lists, menus and other materials so that students will search out meaning and not just begin to crunch numbers.

Notes and textbook references

c) Estimate the results of computations.

A. Students often disregard estimation as a tool for solving problems. They are so quick to get an exact answer. Have students use information provided in charts to estimate the results of computations. You might use information from stock reports, sports statistics, menus, sales flyers or other charts that you create. Ask questions that require estimates instead of exact answers. To encourage the use of estimation, impose time limits or the use of only mental math as a way to solve the problems. Always have students justify their estimates as a part of the discussion of the answers.

Alternative Method
Provide small groups of students with information presented in charts. Have the group write questions pertaining to the information in the charts that should be answered with estimates. Have groups exchange their questions and respond.

B. **Vacation Planner** Have students plan a five-day vacation that covers at least three states. Students will use estimation skills to determine about how much it will cost for their vacation. Allow students to use the internet to obtain information such as hotel rates, gas prices in the various states they plan to visit, menus from restaurants in the areas they will visit, admission prices to attractions, and tax rates. Have students estimate how much gas for the round trip will cost, lodging, food, and entertainment. Students should include tax rates in their estimations. You may have students estimate each day of their trip, each specific expense such as gas, or the trip as a whole.

C. When students are solving problems, they often get caught up in computation. They will continue to add, subtract, multiply, and divide all the numbers in the problem until they cannot do any more operations. Students need to be taught how to look at problems, especially those with decimals and fractions, and come up with an "about" answer.

Notes and textbook references

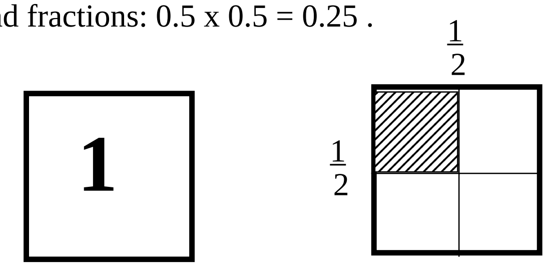
First, the rules of rounding must be firm in the student's mind. Then, the student must be able to have a sense of how much a number can be rounded. This only comes with practice. For example, you would not round 38 to 100. But, you may round 438 to 500 if you are "guesstimating" even though that does not follow the rules of rounding.
Students need to learn the difference between rounding $14.47 to $14.50, to $15.00, or to $20.00. Each one may be used in a different scenario. You may want to give a number like that and have the students come up with reasons why they would use each estimation amount. Once they have a handle on how to round the numbers in the problems, they can then apply the problem solving techniques that they already know to come up with an estimation of the result.

Notes and textbook references

d) Judge the reasonableness of solutions

A. **Models for Judging Reasonableness** (Blackline Master I - 25) Reasonableness of solutions is more than just estimating. Here, we must make sure students understand that when working with fractions and decimals, their old assumptions are not always true. One of the best ways to explain this is to use visual models.

For example, many students have the misconception that when you multiply you get a larger number: 3 x 4 = 12. That is not always true with decimals and fractions: 0.5 x 0.5 = 0.25 .



In addition, students have been misled to believe that when you divide, you get a smaller number. This, also, is not always the case. Consider one-half divided by one-fourth is two. They must understand that you are asking how many one-fourths are there in one-half? See the blackline for transparency sheets to model these operations.

B. Students need to be able to "see the forest" and not just the trees as they work problems. A good way to have students understand what reasonable is would be to start with real-life examples. For example:

1. If your Mom is giving you an allowance, which is more reasonable: \$10 or \$500?

2. If you are buying a shirt, which price is more reasonable: \$40 or -\$40?

3. If you want to describe how tall you are, which is more reasonable: 5 feet or 5cm?

The discussion that would follow questions like these and others would be for the students to understand that they already know what reasonable is – it is what makes sense in terms of real-life. Indicators such as positive or negative, large or small, and type of measurement should be things to pay attention to when we are solving problems, or picking the right answer to a multiple-choice test.

For EOG study this could also be paired together with an error analysis. An error analysis is when you go through the question and determine what error the students made to reach their answer.

For example: John and Jamal went to the movies. John's mom gave him $15.00. Jamal's mom gave him $4.00 more than John's mom. If the movie cost $8.50, how much money does Jamal have after they attend the movie?

A. $2.50
B. $10.50
C. $27.50
D. $32.00

Analysis:

A. The student would have picked A if they did $15.00-$4.00-$8.50. The question is; even without doing the computation; is it reasonable to have $2.50 left? Possibly, so we would not discount this choice right away.
B. This is the right answer; $15.00+$4.00-$8.50. This would also be a reasonable amount to have left and this choice would not be discounted.
C. The student would have picked C if they did $15.00+$4.00+$8.50. Even without doing the computation students should be able to think if it is reasonable that you would have more money left after you had gone to the movies than before you went. This choice would be eliminated.
D. If the student did $4.00 * $8.00 they would have gotten D. Students often multiply the numbers in a problem together if they do not know what to do. However, as in C, they should have been able to eliminate this choice based on the fact that you would not have more money after the movies than what you started with.

After looking at all answers, the student would have two choices left, A and B, and then could do the computation to find the correct answer. If the student knows the mistakes that they will make, they may be more careful to avoid them.

This is an example using a simplistic problem, but can be applied to any multiple-choice problem and extended to more complex problem solving.

Notes and textbook references

Notes and textbook references

C. **Sara's Chocolate** (Blackline Master I - 31)
This blackline can be used as a tool to start the conversation in your class about the reasonableness of numbers and common misconceptions. See 1.02 d A. Encourage your students to come up with their own examples of multiplying and dividing fractions.

D. Students too often accept that when an answer is derived that it is correct. They neglect to think about the reasonableness of the answer. When the reasonableness of the answer is considered in the context of the problem, students may recognize computational errors or rethink the strategy used. To encourage students to think about the reasonableness of an answer, provide the answer to a problem and have students justify, orally or in writing, why the answer does or does not make sense in the context of the problem.

E. (Blackline Masters I -36 and I - 37)
To foster the development of students judging the reasonableness of a solution, allow students to work with the situation cards in groups of three or four. Listening to peers' thought processes and justifications allows students to confirm, reject, or change their own thoughts. On each card a problem situation is provided as well as a solution to the problem. The students should work with the members of their group to:

1. Determine if the answer is reasonable for the situation. No mathematical calculations should be preformed to reach this conclusion.
2. Provide a written argument as to why the answer is or is not reasonable.
3. Provide an appropriate solution if the given solution is deemed to be unreasonable.

1.03 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil.

Notes and textbook references

A. **Draw a Picture —** Have students draw a picture of a word problem. They will often read the problem to find more detail if the assignment is to draw it instead of solve it. After the drawing has been made, have the students show where every number in the problem belongs. Prices go on price tags, distances go on map diagrams, etc. After developing the understanding of the problem required to complete these steps, the students will be better prepared to solve it.

B. **Think-Pair-Share –** Have students read a problem silently and then use think-pair-share to help them determine what to do first. Think-pair-share creates an environment in which every student is expected to think. There is no waiting for the "smart kid" to call out the answer, and every student will be expected to share his ideas. This technique emphasizes the importance of the process in problem solving over the value of getting an answer.

First, the entire class is asked to think silently for a moment about what the first step will be in solving the problem. Next, the students speak softly in pairs to discuss their plans and see whether they agree or can come to an agreement about what a good first step will be. Finally, the pair shares their plan with another pair for more feedback. The group then solves the problem.

C. **Math in Print –** Materials needed: Printed information involving numbers such as sales fliers, transportation schedules, menus, etc. Give students the printed information. Ask the students to work in groups to prepare questions for other students. The group should write two or three questions and make them as challenging as possible. The teacher then collects all questions and prepares a handout for the students to solve in the same groups. The students get one point for each question they answer correctly, and 3 points for each time the questions they wrote stumped other groups. The group with the highest score wins.

Notes and textbook references

D. Devote a bulletin board or wall space to problem solving strategies. You may display all the strategies at the beginning of the year and have students identify which strategy they used when solving a problem or post strategies as students identify that they have used a new strategy.

Some Common Problem Solving Strategies

1. Draw a picture
2. Look for a pattern
3. Guess and check
4. Make a table
5. Make a list
6. Work a simpler problem
7. Work backward
8. Use estimation
9. Use a formula
10. Use logical reasoning