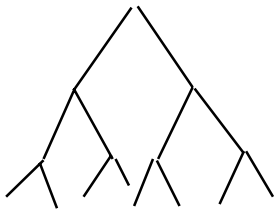
The learner will understand and determine probabilities.

4

4.01 Develop fluency with counting strategies to determine the sample space for an event. Include lists, tree diagrams, frequency distribution tables, permutations, combinations, and the Fundamental Counting Principle.

Notes and textbook references

A. **Permutations** Assign partners. Give each pair of students pattern blocks in different shapes. Students can challenge each other to make permutations of multiple shapes. Students can then draw the shapes to list all the different permutations that can be made.



B. **Tree Diagrams and the Fundamental Counting Principle** (Blackline Masters IV - 2 and IV - 3) This activity explains how the outcomes of a multi-step process can be found by the Fundamental Counting Principle and by a tree diagram. Teachers can point out that the the number of paths on a tree diagram comes from the number of branches created by the first decision multipled by the number of branches created by the second decision. This helps explain why the Fundamental Counting Principle works as it does.

Students are asked to create a new tree diagram for a second problem and then use it to answer probability questions.

C. **My Word** (Blackline Masters IV - 4 and IV - 5)
This activity describes an experiment in which letters are generated by using a spinner and a coin. A tree diagram may be used to determine the possible outcomes. Once all the outcomes are discovered, the students can answer probability questions and compare theoretical results to experimental ones. .

Notes and textbook references

D. **Frequency Distribution** (Blackline Master IV - 6)

Materials: 20 - 25 strands of spaghetti pasta, metric ruler with measurements to nearest mm, recording sheet.

Students are asked to break each spaghetti strand into two pieces. They should not be encouraged to break them exactly in half. Then the students measure each piece to the nearest mm and record the length with a tally mark in the frequency table.

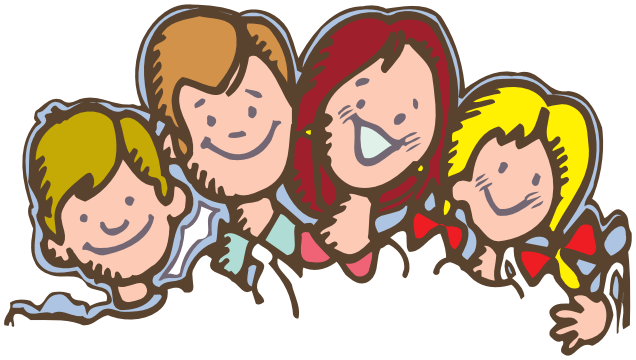
E. **Combinations** Arrange students in groups of four. Have groups do the following:

1. Predict how many different ways their group members can be arranged.

2. Form a line of your group members. Then form as many different lines as you can. How many could you form?

3. Simulate the problem. Represent each member with an object, such as a pencil, book, or a shape. Arrange the objects in as many different lines as you can. Do you get the same result as in Question 2?

4. Represent each group member with a letter or number. Make an organized list of all the possible arrangements.



4.02 Use a sample space to determine the probability of an event.

Notes and textbook references

A. What's A Sample Space? The teacher will use props to assist students in their understanding of sample space and probability.
The teacher can begin by explaining to students that the sample space is the set of all possible outcomes for an experiment.
Use a coin to explain that the sample space would be heads and tails, {H, T}. For a die, the sample space would be {1,2,3,4,5,6}.

Probability is written as the number of desired outcomes over the number of outcomes in the sample space. For example if the desired outcome on the throw of a die is a number great than 3, {4, 5, 6} then the probability would be $\frac{3}{6}$ or $\frac{1}{2}$.

Possible class activities for sample space.
Have the students determine the sample space and probability for the following situations or experiments.

1) Having someone from your math class selected to be 6th grade student of the month.

2) Look at a local menu and determine the probability of ordering a meal with chicken.

3) Your school is having a kickball tournament. Make a sample space of all the sixth grade classes you could compete with.

B. What's the Probability? (Blackline Masters IV - 11, IV - 30, and IV - 34)
Students will work in pairs to discover how increasing the sample space changes the probability of an event. The students will use two spinners, one with four numbers and one with eight numbers. The students will begin by identifying the sample space for each spinner. They will then chart the probability of the following outcomes; spinning a 1, spinning an even number, spinning a number greater than 5, spinning an odd number, spinning a prime number. After students have completed the chart, discuss how the sample size affected the probability.

Notes and textbook references

C. High Rollers (Blackline Master IV - 31)

Students will work in pairs to chart the sample space for two dice.
(1,1) (1,2) etc.,etc.
After completing the sample space (36 total), the students should chart the probability of the following outcomes:

1. The probability of rolling an even sum.
2. The probability of rolling a 2 on one die.
3. The probability of rolling the sum of 7.
4. The probability of rolling a sum that is a prime number.
5. The probability of rolling a sum less than ten.

D. Go for the Gold. (Blackline Master IV - 32 and Blackline Master IV - 33)

Students will practice using probability and sample space as they race for the gold in this game of probability. Students can play in groups of 2-4. Each student begins at start and goes to the first open space. The student reads the instructions on the space and chooses which spinner to use based on their understanding of sample and probability. Play progresses in a clockwise direction and the winner is the first one to reach the finish.

E. Sweet Probability Give students a small bag of M&M™s or Skittles™ candy. Have each student open their candy (please don't eat until after the lesson) and determine the sample space for their bag. Have a class discussion about the probability of choosing a certain color of candy.
Possible examples:

- The probability of choosing a red
- The probability of choosing an orange
- The probability of choosing a candy that is not green

Discuss why students sample spaces may differ and also why the probability differs from student to student.

4.03 Conduct experiments involving simple and compound events.

Notes and textbook references

A. Color Cubes Give each pair of students a paper lunch bag containing 15 unifix cubes, (5 white, 5 yellow and 5 blue). Have the students choose a cube 20 times and record the results on a piece of paper. Then have them determine their probability of selecting each color. Discuss the how students' findings differ and why.

B. Paper Basketball Students will work in small groups to conduct an experiment with free throws. Each student will use a notebook sheet to make a paper ball. Each student will have an opportunity to shoot their ball 10 times to try and hit the wastebasket. After each team completes their free throws, they will choose the player from their team that has the highest probability of beating other teams. The chosen player from each team will then compete against the other team representatives. The winner is the player who gets the most points in the final round.
Materials needed: waste basket for each group, paper for balls

C. Sweet Success Give each pair of students a paper bag with 10 mini lollipops or candies. Have the students list the sample space and then select one candy 15 times. Have them make a list of the candies that were chosen and then write the probability of choosing each type of candy. Discuss the differences in the probabilities.
(After you're done, they can choose a piece of candy and enjoy.)

D. Spinner and Dice Give each pair of students a coin and a spinner. Students should list the sample space of the spinner and coin. (Example: H,1; H, 2; H, 3; etc.) Have the students spin the spinner and toss the coin 25 times. They should chart their results and then list the probability of each outcome.

E. The Messy Sock Drawer Tory has a messy sock drawer. It contains 7 black socks, 4 blue socks and 9 white socks. On a particular morning he reaches in and picks out two socks without looking. What are his chances of getting a matching pair?
(Students can use color cubes or color paper cubes to represent the socks) Have the students identify the sample space and then choose one sock and without replacing it choose a second to see how many times they have to choose before matching a pair. Have them chart what they pick each try until they get a matching pair. Discuss students findings as a class.

Notes and textbook references

4.04 Determine and compare experimental and theoretical probabilities for simple and compound events.

A. **Making Spinners** (Blackline Master IV - 11)

Give each student a circle divided into eighths. Ask the students to color their circles using three colors. Ask the students to record the probability of each color by counting it as a fraction. Using the circles, make a spinner, spin 20 times and record the results. On the back of their circles, ask the class to make a new circle using four colors, coloring one-fourth of each circle one color. Have each student (using a pencil and paper clip) spin the spinner 20 times. Compile the results and compare them to the expected results.

B. **Free Throw Percentages** Free throw percentages can be interpreted as a experimenal (empirical) probability. For instance, if a player hits 90% of his free throws, this means that a good estimate of his probability of being successful on his next shot is 90%. Have students keep data on a favorite basketball player. Does his free throw percentage actually equal his probability of being successful when attempting a free throw?

C. **Space Race** (Blackline Master IV - 7)

Materials : A pair of dice. Students follow instructions to bet on a space race. They should discover that space ship number one has no chance of progressing, and that it is much more likely that ships numbered six, seven, and eight will come in first. The table below illustrates why these numbers are rolled more often.

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Notes and textbook references

D. Application Contingency (Blackline Master IV - 8) Students are provided with age and gender data for applicants to join a space colony. Students will apply the definition of simple probability to answer the questions about these applicants. Note: A contingency table displays data in a table that covers all possibilities, all contingencies. In this case, the applicants must be male or female, and their ages must fall into one of the categories. No applicants were allowed below the age of 20.

E. Mini Review – Probability (Blackline Masters IV – 9 and IV – 10) This mini review covers most of the concepts of probability from this section. Students should be allowed to work in pairs to share strategies.

F. Dice Probability How often does the number 1 come up when a number cube (die) is rolled? Compare data when a student rolls 10 times, when 6 students roll 10 times, and when everyone in class rolls 10 times. Which set gives probability closest to $\frac{1}{6}$?

G. Cube Probability Place 2 red cubes, 2 yellow cubes, and 2 brown cubes in a paper bag. Draw, without looking, one cube from the bag 12 times. (Be sure to place the cube back into the bag after every draw.) Record results. Compare results when four students' draws are combined; 8 students' draws are combined; and the entire class.

Notes and textbook references

H. Sample Size Place six plastic colored eggs in a large brown bag. Have students draw eggs out of bag one at a time and record result (do this a total of 20 times). Be sure to put the egg back into the bag after each draw. Next, place students in groups of five to six and repeat the above procedure. Be sure to record results. As a whole class, examine data collected. Predict what the results might be as the sample size increases. For example: "What do you think the results would be if each sixth grade class were involved?" Compile the report data. Make an "official" presentation with charts and overheads to another class, principal, etc.

I. Family of Probability Suppose a family has four children. What are the possible birth orders?
What are the chances that a girl will be the oldest?
If there are three or five children, what is the probability that a boy will be the middle child? Explore the probabilities of the families in your class.

J. Are Spinners Really Random? (Blackline Master IV - 12) Use the blackline master provided to test spinners for randomness. Use a paper clip as a spinner arrow. The spinner provided can be used to test for four, five, or six outcomes. Students should select which dial on the spinner they will test and then work in pairs to spin the spinner 120 times. They will record the actual outcomes and compare with the expected outcome. Students should discuss any differences and brainstorm as to why those discrepancies may have occurred. Were the outcomes drastically different throughout the class? Is the average of all class outcomes closer to the expected outcomes?

Notes and textbook references

K. Estimating Wildlife Populations (Blackline Master IV - 13)

Materials: Paper bags, marbles, cubes, or candies of the same shape and size in two colors. Halloween taffy in black and orange wrappers works well.

Directions: Prepare a paper bag for each group with a number of marbles or candies inside. The students should not know how many items are in the bag. These represent the number of trout in a pond. Have the students add a known number of similar items to the bag. For example, the bag may have been prepared with 72 (unknown to student) red marbles and they can add 20 blue marbles. This process represents the wildlife manager putting tagged fish into the pond. The students should close the bag and mix the items. They then do several trials of taking a random sample from the bag by dipping a half-cup measure into the bag and removing the marbles they "catch." This corresponds to the wildlife manager catching a sample of fish. On each trial the students calculate the percentage of tagged fish in the sample. They should conduct several trials and average the results. This average serves as an experimental probability of finding a tagged animal.

$$P(\text{catching a tagged fish}) = \frac{\text{Number of tagged fish in the pond}}{\text{Total number of fish in the pond}}$$

Since the students have values for the probability of catching a tagged fish and the number of tagged fish, they can calculate the number of total fish in the pond.

Notes and textbook references

L. Spin to Win! (Blackline Masters IV - 14 and IV - 15)

In playing this game, students compare expected to experimental outcomes in three different ways. They calculate the outcomes and probabilities of a single spin, they calculate the probabilities of spinning more than, less than, or equal to 50 cents, and they predict the outcome of playing a carnival game. The expected value of playing this game is the average of all 12 outcomes, a bit more than 58 cents. In spite of the fact that students will win more than 55 cents less than 25% of the time, the expectation is for the player to win in the long run. This is a good activity to bring out the law of large numbers.

HINT: An easy way to make the pie chart is to use a Hefty™ plastic plate. These plates have 36 dimples around the rim to correspond to the 36 trials in this game.

4.05. Determine and compare experimental and theoretical probabilities for independent and dependent events.

Notes and textbook references

A. Losing Your Marbles (Blackline Master IV - 16)

Students use coins to simulate marbles falling through a maze. They use probabilities of independent coin tosses to determine a theoretical result for the experiment.

For a marble to fall into the far right slot, the marble must fall right each time. There are four choice points and at each one, the marble has a probability of 0.5 to fall right. The probability of falling into the far right slot is $(0.5)^4 = \frac{1}{16}$

B. Oops! (Blackline Masters IV - 17 through IV - 24)

Materials: Playing cards printed on transparency film, cut apart, and placed in a bag, a playing piece for each team, paper clip for a spinner, game board printed on transparency film, overhead projector.

Directions: The leader divides the class into two teams and decides which team will go first. On a team's turn, the leader places one of the playing cards on the game board. If the team can answer correctly, the leader spins the spinner to find out how much the team advances. The arrows on the board indicate that a team will slide forward or go backward in the direction of the arrow. The first team to reach the finish wins. Note: As students become more proficient in calculating these probabilities, you may wish to allow the students to play in small groups. In this case, the playing cards will need to be printed on card stock. Answers are provided for use in this situation.

Notes and textbook references

C. Fraction Cubes and Probability (Blackline Masters IV - 25 through IV - 27)

Materials : unmarked wooden cubes (or spinners), pens, recording sheet. A chart is provided in which fractions are generated with a cube that produces numerators and one that produces denominators. The resulting possible fractions are shown in the table. Students use the results of the table to answer probability questions.

Directions: Students mark the blank cubes so that the numerator cube contains numbers 5 - 10, and the denominator cube contains numbers 5, 8, 10, 12, 17, 20. Students complete the table to produce the possible fractions. They answer theoretical probability questions based on the data in the table. Then the students actually roll the dice and record the resulting fractions. They compare the experimental results with the predicted probabilities.

D. Modeling Dependent and Independent Events

Materials: Five white marbles, one red marble and a paper bag.
Activity: (Your part) Suppose you put five white marbles and one red marble in a paper bag. You reach in without looking to draw a marble, look at it, and put it back in the bag. A second person draws a marble. The probability that the second person will draw the red marble is the same as it was for you because the bag contains the same five white and one red marble. The two draws are independent events. However, if you keep your marble when you draw it, the probability that the second person will draw the red marble is different because there is one fewer marble in the bag. The two draws now are dependent events.

(Students part) Organize students into groups. Give each group a coin, a bag of marbles, a number cube, a stack of cards, and a blank spinner. Have the students use any of these items to create two problems: one that involves two independent events and one that involves two dependent events.

4.06 Design and conduct experiments or surveys to solve problems, report and analyze results.

Notes and textbook references

A. Planet Collector Cards (Blackline Master IV – 28 and IV – 29) Students use the spinner to simulate buying Captain Krypton Cereal in the attempt to get an entire collection of Planet Collector Cards. Each student should conduct the experiment three times. Then data from the entire class can be gathered to determine how many boxes of cereal one might expect to buy to get the entire set.

B. Counters and Cups

Materials: 3 two-colored counters and a cup for each group

Activity: Use cups and counters to explore the experimental probability that at least two of three children in a family are girls.

Step 1: Place the three counters in a cup and toss them onto your desk.

Step 2: Count the number of red counters. This represents the number of boys. The number of yellow counters represents the number of girls.

Step 3: Record the results in a table like the one shown.

Trial	Outcome
1	B B G
2	B G G
3	
50	

Step 4: Repeat steps 1-3 for 50 trials.

Using a class chart report results and discuss the results. Students then could explore the experimental probability that two of five children in a family are boys.

Notes and textbook references

C. Lottery Pick This activity will divide the class into three groups. Each group will perform their activity and record the results.

Lottery — Pick 3

Purpose: To understand the probability of independent events.

Activity: Three containers each contain 10 balls. The balls in each container are numbered from 1 to 10. A person draws one ball from each container to determine the winner. Group discussion: 1) What is the probability of the number 111 being drawn? 2) If the first number drawn is 1, what is the probability of the next two numbers also being 1? 3) If the first two numbers drawn are 1, what is the probability of the third number also being drawn?

Lottery — Pick 4

Purpose: To understand the probability of independent events.

Activity: Four containers each contain 10 balls. The balls in each container are numbered from 1 to 10. A person draws one ball from each container to determine the winner. Group discussion: 1) What is the probability of the number 9876 being drawn? 2) If the first number drawn is 9, what is the probability of the next three numbers being 876? 3) If the first two numbers drawn are 9 and 8, what is the probability of the next numbers being 7 and 6?

Lottery — Pick 6

Purpose: To gain an understanding of the probability of dependent events.

Activity: A can contains 20 balls numbered from 1 to 20. A person draws six numbers without putting the number back. Group discussion: What is the probability of the numbers 2, 4, 6, 8, 10, and 12 being drawn?

Come together as a class and discuss the difference between independent and dependent events. Allow the groups to share their results. Ask: Which lottery, Pick 3, Pick 4, or Pick 6 would they play first and why?

D. Rolling Number Cubes

Notes and textbook references

Materials: Enough number cubes for your groups to have 2 each.

Activity: This game involves rolling the two number cubes as often as you want. After each roll, the two numbers on the number cubes are added. The purpose of the game is to come closest to the number 25 without going over. You must always roll both number cubes and add the numbers. Have groups record their results in a table.

Roll	Numbers rolled	Sum
1		
2		
3		
4		
5		

You have rolled three times. As a group discuss:

1) Improving your score (remember, scores over 25 are losers).

2) Rolling exactly 25.

3) Exceeding 25.

Come together as a class and discuss results.

Notes and textbook references

E. Draw it Out

Materials: A bag, hat, or box and 4 black beads and 3 red beads for each group.
Activity: In a hat, bag, or box there are 4 black beads and 3 red beads. One bead is drawn and replaced. Then a second bead is drawn.

What is the probability of drawing:

1. 2 black beads?

2. 2 red beads?

3. A black and a red bead?

Students should record and discuss results as a class.

F. Decimal Drop (Blackline Master IV - 1)

Materials : Meter stick, tag board strip 50 centimeters long, marker.
Procedure: Each group should mark their strip in decimeters showing the positions of 1 – 5 decimeters. Have one student stand on a chair and hold the strip vertically. Another student holds his thumb and forefinger at the bottom of the strip. His fingers are not touching the strip, but should be in position to try to catch it. The student on the chair will drop the strip and at the same time, the student on the floor will attempt to catch it using only his thumb and forefinger. The group will estimate where the point of capture occurred. This process is repeated three times for each person in the group. It will probably occur that most students capture within the same decimeter range. They will now mark off these intervals to the nearest tenth of a decimeter (centimeter) and repeat the process recording the results to the nearest centimeter. Class data can be used to determine the mean and the median.