The learner will understand and use properties and relationships of plane figures.

3

3.01 Identify, define, describe, and accurately represent triangles, quadrilaterals, and other polygons.

In the 1950's, Dina and Pierre Van Hiele, high school teachers in the Netherlands, developed levels of learning and reasoning in geometry. Their research supports the sequential, activity-based learning of geometry.

A. Using the Blackline Masters II - 5 through II - 7 for triangles, have students work as partners to sort the triangles in as many different ways as they can. Each pair should keep a list of their sorting. As a total group, discuss and model the classifications. How many triangles could belong to more than one category? If the triangles are labeled, this is a good exercise for a Venn diagram.

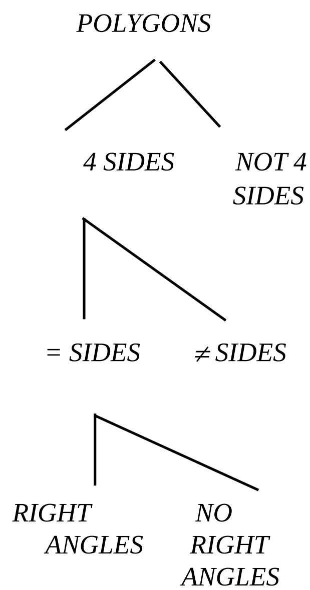
B. Have students put together pattern block pieces to make as many quadrilaterals as they can. Have them name and describe the quadrilaterals, determine ones that are congruent, and discuss similarities and differences. Have them repeat this activity with pentagons, hexagons, and other polygons.

C. Have students follow the directions for creating triangles on the Blackline Master III - 10, "Triangles" . They will need different colors of chenille stems or pipe cleaners, coffee stirrers, rulers, and scissors. Remind the students to cut the coffee stirrers and thread them onto the chenille stems to complete the activity.

D. Repeat the above activity to create quadrilaterals. Use Blackline Master III - 24.

Notes and textbook references

E. Using graph paper (Blackline Masters III - 1 through III - 6) have students draw a variety of triangles (right, obtuse, scalene, isosceles, equilateral), quadrilaterals, (parallelogram, rectangle, square, rhombus, kite, trapezoid, isosceles trapezoid); pentagons, hexagons, octagons (regular and not regular). Students should label and classify the polygons according to type.



F. Use recycled boxes, cartons, hose containers, etc., to build a model of a futuristic community. Draw a map of the model focusing on the two dimensional shapes and angles required to symbolize the solids.

3.02 Make and test conjectures about polygons involving:

Notes and textbook references

a. Sums of the measures of interior angles.

A. Distribute many different kinds of triangles and quadrilaterals to students. (Blackline Masters II - 5 through II - 10) Direct them to create a table with labels for the name of the shape, the measure of the angles, and the sum of the interior angles. Have students number the angles, measure the angles using their protractors, and then record their data in the table. Tell them to look for and discuss any relationships they see. They should soon discover that the sum of the interior angles of a triangle is 180°, and the sum of the interior angles of a quadrilateral is 360°. A variation of this would be to have the students draw many different triangles and quadrilaterals and use these for the investigation instead of providing them for the students.

B. Ask students to select a particular polygon. (Blackline Masters II - 5 through II - 10 and III - 7 through - 9) Have them draw at least five different examples of their polygon. Direct them to measure the angles of each polygon, organize their information, and then draw conclusions about the measure of the polygon's interior angles.

C. Using a paper cutter, cut a variety of triangles out of construction paper. Don't take a lot of time trying to make isosceles or any particular type. Pass them out so that each student has at least one. If your students are working in groups, make sure they have several different triangles.

Display a triangle on the overhead. Name the three vertices with letters and ask students to name each of the interior angles of the triangle. Then ask them to estimate the size of each angle. Record the best estimate for each angle.

Based on their work with measuring angles, ask students to work with a partner to estimate the angle size of each angle in the triangles they have. Give time for discussion and collaboration. Call the class back together and

Notes and textbook references

ask for several volunteers to share their estimates. As they do, record on the board or overhead, and ask the class to use calculators to find the sum. Record the sums. They may show a pattern, and they may not, but it gets the kids thinking about the sum. If they are all close to 180° this is a great assessment of students ability to estimate angle size.

Now for the fun. Give out glue sticks and construction paper in colors contrasting to the triangles. Show students how to rip the three angles apart, rotate them so all vertices are lined up. What kind of angle have we made? How many degrees are in a straight angle? What should our estimates have added up to?

Students glue their angles down, and write under them with markers, Angle A + angle D + angle T = 180°. You may ask students to use protractors to measure and verify, but they should see that they all create a straight angle. Display so students can see that all the different triangles still have the same result.

D. Randomly cut quadrilaterals for students and several large ones for demonstration.

Show a large, irregular quadrilateral to the class. We know the interior angles of triangles always add up to the same amount. How many degrees? What do you think about the angles of four sided polygons? Will the sum of the angles always be the same? If not, why not? If so, why and what would the sum be? Give students time to work in pairs or small groups and discuss this. Ask for responses.

Students may have lots of ways to think about this. If they need help, ask what happens if you cut or fold a quadrilateral on a diagonal. (Be careful not to say "in half", which I have done, because a student may have a very odd shaped figure and can show that the areas of the two resulting triangles are not equal.) Students can fold their shapes to make two triangles. If we know the sum of the angles of one triangle is 180° , what would be the sum of the angles of two triangles? How would you find a missing angle here?

E. After completing 3.02a) D, have students continue their exploration using other polygons. See Blackline Masters III - 24 through III - 28.

b.) Lengths of sides and diagonals.

A. Prepare diagonal strips by copying centimeter grid paper (Blackline Master III - 2) on card stock or other heavy paper. Two sheets will be enough for a class. Cut strips 11 cm long and 1 cm wide. Have students punch a hole in the middle cm square with a single hole punch. Join the diagonal strips with a brass fastener. On each end of the strips, mark a dot at the midpoint of the cm square.

Students work in pairs. Make an “X” with the diagonal strips. Place on a piece of plain paper and hold in place. With a pencil, make a dot on the paper at the midpoint of both ends of each diagonal. Move the strips aside and connect the dots at the vertices to make a quadrilateral. Label the type of quadrilateral. (rectangle) Make another “X” with the diagonal strips at a different angle and repeat. Label the new shape. (rectangle) Repeat this activity several times.

Discuss with the class what they discovered. Ask if anyone drew a shape with the diagonals perpendicular to each other. What shape do you get? If no one tried this, ask students to think about a prediction. Then have partners try it out.

In this activity, the diagonals are congruent and bisect each other. If these conditions exist, will the shape always be a rectangle? Why? What additional condition must be present to get a square?

Think about how many different combinations of conditions there are. In his book, Elementary and Middle School Mathematics, John Van de Walle states that any quadrilateral can be determined by its diagonals.

Ask students to use the strips from the previous days, adding more holes if they wish, to see what other shapes they can make. It is not necessary for students to try all possible combinations, or discover the rules for every quadrilateral.

Part Two:

The next day, begin by reminding the students that yesterday they worked with congruent diagonals that bisected each other. Ask what they think would happen if the diagonals weren’t congruent. Allow discussion and speculation, even arguments, without providing any answers.

Notes and textbook references

Making Conjectures
Whenever students make attempts to extend patterns, they are making mathematical conjectures. They can learn to make conjectures by asking what if ? questions. Asking students to justify solutions will also help them make conjectures. from Navigating through Algebra in Grades 3-5

Notes and textbook references

Give students a new diagonal strip, say 5 cm. Again hole punch the middle cm and connect it to one 11 cm strip with the brass fastener. Repeat the process used yesterday. What shapes do you get? (parallelogram) Will that always be true with non-congruent diagonals that bisect each other? Did anyone try it with perpendicular diagonals? If no one did, discuss what students think might happen. Try it out. It will generate a special type of parallelogram, namely a rhombus. All sides are equal in length if the diagonals bisect each other and are perpendicular.

Part Three:

Begin the next class with a review of the past two days. Then ask students to help you think about all the possible ways diagonals of quadrilaterals can be related to each other; all the conditions that are possible. You might begin a chart something like this on chart paper or see Blackline Master II - 12.

Polygon	**Diagonals**	**Angle of intersection**	**Do they bisect each other?**
	Congruent	Not perpendicular	
	Not congruent	Perpendicular	

B. **Discovering Diagonals I** Have students use rulers and geoboards to complete the activities on Blackline Masters III - 17 and III - 18. Have students make the given polygons on their geoboards to investigate diagonals. As they make the diagonals, have the students record the results on their Blackline Masters. Give students time to talk with their peers about their discoveries. Have students write conclusions about their findings and share them with a neighbor. *Adapted from Math Matters 2004.*

C. Using the Blackline Master **Discovering Diagonals II**, (Blackline Master III - 19) have students draw other figures and find the number of diagonals in each (triangle, quadrilateral, pentagon, hexagon, heptagon, octagon). Students may wish to use a different color for each diagonal to make counting easier. Create a chart to organize the information the headings "Polygon," "Number of Sides/Vertices," and "Number of Diagonals." Have students study the patterns in the chart and create conjectures to predict the number of diagonals in any polygon. Test the conjectures to develop a generalization relating the number of sides to the number of diagonals.

Notes and textbook references

Notes and textbook references

c.) Parallelism and perpendicularity of sides and diagonals.

A. Diagonals of Parallelograms Review the definition of a parallelogram … a quadrilateral with two sets of opposite sides parallel and congruent.

Work first with parallelograms with one set of sides longer than the other, and with no right angles. Give students isometric dot paper, available in the Blackline Masters III - 13. Ask students to draw at least 4 different parallelograms. Discuss how they are alike and how they are different, recording on chart paper of overhead. The parallel nature of opposite sides, the equal opposite angles and sides, are all properties of a parallelogram.

There is another property of polygons that we can explore: two diagonals. Ask students to discuss with a partner or group what the diagonals will look like. Predict the answers to the following questions (this is the conjecture part) :

1) Will the two diagonals be congruent?
2) Will the diagonals be parallel?
3) Will the diagonals be perpendicular?
4) Will the diagonals bisect each other?
5) Will the answers to these questions be the same for any parallelogram?

Now have students draw both diagonals for each parallelogram. Test their predictions/conjectures with others in their groups. Discuss each question, especially the last. How could you prove the answers would or would not be true?
Homework: Using more iso-paper, draw more parallelograms and answer the questions for each.

B. Working with Special Parallelograms A rhombus is a special kind of parallelogram. It has opposite sides parallel and equal, but all four sides are congruent. The blue and tan rhombuses in the pattern block sets will show this. Repeat the process above making conjectures about questions 1-5, then drawing the diagonals to test the conjectures. Which question has a different answer?

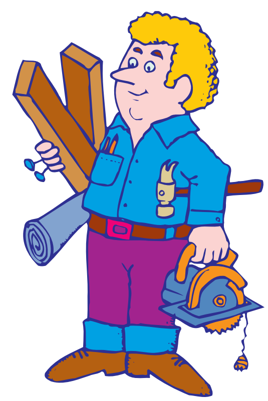
Now, try the same activities with a square.
Draw, predict, test. A square is a special kind of parallelogram, and a special kind of rhombus. It has opposite sides parallel and equal, and has four right angles.
Again have students draw several different squares. Use centimeter graph paper for this. (Blackline Master III - 2) Again ask students to describe likenesses and differences. The properties of squares are very specific. All the squares will look the same except for size (they are similar), while the other parallelograms will vary in shape according to length of sides and size of angles. Again, ask students to make conjectures about the diagonals.

1 Will the two diagonals be congruent?
2 Will the diagonals be parallel?
3 Will the diagonals be perpendicular?
4 Will the diagonals bisect each other?

Will the answers to these questions be the same for any square?
Discuss the children's thinking about these questions.

C. Give groups of students identical sets of tinker toys. Include circles, sticks, and plastic/paper pieces. Allow them to label the wheels (points) with circle dots and markers or masking tape. The task is to build something and then list the geometric concepts that are modeled.

Notes and textbook references



Notes and textbook references

3.03 Classify plane figures according to types of symmetry (line, rotational).

A. Have students explore line and rotational symmetry by making models. Create congruent copies of several large shapes (square, rectangle, isosceles trapezoid, equilateral triangle, right triangle) using different colors of construction paper. Connect each pair of shapes through the center using brads. Discuss the lines of symmetry (if any) in each figure. Summarize on a chart. One figure at a time, mark one vertex with an arrow or other symbol to easily track its movement. Have the students rotate the top figure to find the number of places in which the figure exhibits rotational symmetry. Add to the chart. Have the students develop a definition of rotational symmetry.

B. Using pattern blocks, have students explore rotational symmetry in the same manner as above. Have students mark one vertex, and rotate the figure over an identical piece to find rotational symmetry.

C. After students have explored rotational symmetry in concrete objects, have them identify items in the classroom that may exhibit rotational symmetry. Have students explain their thinking, and test the figures, if possible.

D. Have students work in pairs to cut out the shapes in the Blackline Masters III - 14 and III - 15. Students should draw a Venn diagram and label one part “Line Symmetry” and the other “Rotational Symmetry.” Students should first predict which shapes will show line symmetry, rotational symmetry, or both. Students then test their predictions, and glue the shapes in the appropriate areas of the Venn diagram. The teacher may wish to prepare a diagram on chart paper and to have the shapes already cut out to have volunteers check their work by placing the shapes in the appropriate areas of the class diagram.

Notes and textbook references

E. Use the following as an introduction to rotational symmetry. Locate enough rectangular and square boxes with lids so each pair of students has one of each. Before distributing the boxes to students, place a mark on the inside of each lid near the midpoint of one of the edges. Place another mark directly below the mark on the inside bottom of each of the boxes. The purpose of the marks is to help the students remember the original position of the lids. Begin with the rectangular box and have students lift the lid from the box. Ask students to predict if they think the lid could be turned in such a way to fit on the box as it did in the original position. Have students turn the lid and ask volunteers to demonstrate and describe how far the lid was turned (180˚; halfway around). Tell students the rectangular lid is said to have half-turn, or 180-degree rotational symmetry. Repeat with the square box. Students should discover that the square lid has quarter-turn (90˚), half-turn (180˚), and three-quarters turn (270˚) rotational symmetry.

F. Have students investigate company logos/trademarks for line and rotational symmetry. Have them identify whether the logos/trademarks have line and/or rotational symmetry. They could be asked to describe the type of symmetry they find in the figures or to compare/contrast the symmetry in two different logos/trademarks. Have the students go on a scavenger hunt to find a logo/trademark that has rotational symmetry but no line symmetry, or a logo/trademark that has line symmetry but no rotational symmetry.

G. As a way to connect social studies and mathematics, have students investigate state and/or national flags for line and/or rotational symmetry.

Notes and textbook references

H. A plane figure has rotational symmetry if it can be rotated about a point less than 360˚ and match exactly the original shape. Typically the figure will match if rotated 90˚, 180˚ , 270˚, or 180˚. Some polygons will have even more degrees of rotational symmetry.

Have students work in groups. Each cuts out a different plane figure from Blackline Master III - 14 and III - 15. These can be run on card stock or construction paper. Students place the shape on plain paper and trace around it to make a copy of the shape. Then they slowly rotate the shape to see if it fits the shape when turned. Students should record the name of their shapes, and if and when they 'fit' the traced shape. Rectangles that are not squares will fit when turned 180˚, for example. An equilateral triangle will match at 120˚ and 240˚. Ask students to share in their group what happened with their shape.

After students have worked to explore in groups, bring the class together for discussion. Ask students to share which shapes they worked with, and what they discovered. Record conclusions. After several figures have been recorded, ask about any patterns students see.

I. **Going Logo for Symmetry** in Navigating through Geometry from NCTM addresses this objective well. If you don't have this resource, you can order it on-line from NCTM.org/catalog. The lesson includes really good information about rotational symmetry and the background knowledge students need for this objective.

3.04 Solve problems involving the properties of triangles, quadrilaterals, and other polygons.

Notes and textbook references

a.) Sum of the measures of interior angles.

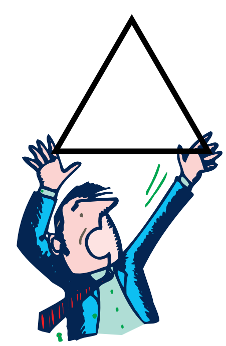
A. Have students solve "testing" problems in pairs, displaying their solutions on newsprint or drawing paper. Collect 4-8 problems per class, giving each pair one problem. Have the class work out the solutions, then, once the solution is verified, have them display them to share with the class.

Possible problems:

1. Mike is building a picture frame in the shape of a quadrilateral. If he has already made three corners each with angles of 90°, what angle measurement must he use to create the last angle?

2. If Mike then built a picture frame in the shape of a triangle and used the measurement of 50° for the first two angles, to what angle measurement must he cut the remaining angle?

B. Have each student draw or cut out three different triangles. Ask the students to color the corners of each triangle the same color and then to tear off the corners. They will paste them down side by side so each corner touches at the same point. Have the students compare their triangles with each other and talk about what they observe. To conclude the activity, ask students to write about the sum of the angles of all different kinds of triangles.



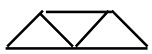
C. Use the patterns for triangles in the Blackline Masters II - 5 through II - 7. If possible, duplicate a set on stiff paper for every two students. These should be saved for other activities. Use the different triangles to demonstrate that the sum of the angles of a triangle is 180°.

Notes and textbook references

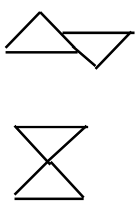
D. Before class, cut out a variety of triangles. Label six triangles A to F for *each* group of students. Ask individual students to make a chart like this:

Polygon	Degrees in angle1	Degrees in angle 2	Degrees in angle 3	Sum of the angles
A				
B				
C				
D				
E				
F				

Ask students to measure the three angles in each triangle and complete the chart. Why is measuring angles very difficult? Talk about extending the rays of an angle to make measuring easier. See Blackline Masters II - 3 and II - 5 through II - 7.



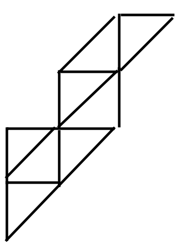
Yes



No

E. Begin collecting rich problems and investigations that require the use of geometric concepts. Here are a few beginning ideas:

OCTRI-OMINO



<u>Octri-ominoes</u> - How many different plane figures - polygons can be made with 8 equilateral triangles? Each triangle must share a complete side with at least one other triangle. These shapes will be called “octri-ominoes” because they are formed from 8 triangles. Don’t bother to look this up in the dictionary; it is an invented word. Students will find it easier to record these shapes on triangular dot paper; many sources include this as a blackline master for the pattern block triangle. See Blackline Master III - 13 or III - 16.
Students may want to begin this investigation with fewer triangles; for example, 3, and search for all the Triominoes, or 4 and search for all the Tetratri-ominoes, etc. By keeping track of data in a table, students may discover a pattern that will help them determine when they have found all the Octri-ominoes.
The next part of this investigation is finding out which Octri-ominoes fold into an octahedron. An octahedron is a polyhedron with 8 faces, triangular faces in this case. Could this same idea be used with other polygons?

Notes and textbook references

F. Repeat activities B and C with quadrilaterals. Be sure to include all types: rectangles, squares, kites, parallelograms, rhombuses, trapezoids, and irregular figures. See Blackline Masters II - 8 through II - 10.

G. Have students create these figures. Ask them to predict when their designs will be alike and when they will be different.

1. Draw a pentagon with angles of 90^o, 15^o, and 20^o . What are the measures of the other two interior angles?
2. Draw a triangle with angles of 40^o and 55^o. What is the measure of the other angle?
3. Draw a quadrilateral with angles of 50^o and 80^o. What are the measures of the other two angles?
4. Draw a quadrilateral with angles of 205^o and 15^o. What are the measures of the other two angles?
5. Write directions of your own for another student to solve.

H. Research the Wright Brothers' first flying machine, the glider. Create a replica of the glider. Identify geometric terms or properties as related to the glider. Explain why Kitty Hawk was chosen as a good location for the flight.

Notes and textbook references

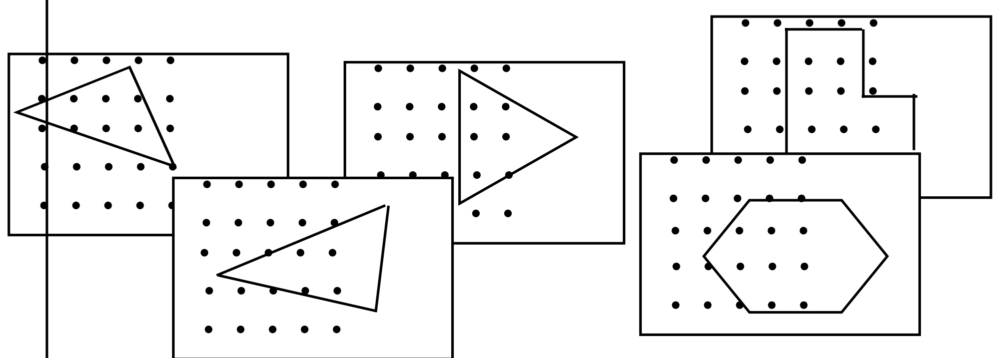
b.) Lengths of sides and diagonals.

A. **Exploring Polygons** Have students explore polygons on a geoboard or draw on paper. Students should measure the length of the sides in cm and then measure the length of the diagonals. Make conjectures while looking at patterns on the completed chart (Blackline Master III - 21). Be sure to give the students three choices for statements to use to complete the chart.

1. The length of the diagonals is congruent to the length of the sides.
2. The length of the diagonals is longer then the length of the sides.
3. The length of the diagonals is shorter than the length of the sides.

Students can work in small groups to discuss finding and solutions. Then the groups can share with the class by having students demonstrating their findings on the overhead geoboard.

** If students try irregular polygons, they will have multiple lengths for the sides and diagonal measurements.



B. **Perfectly Square** Students make squares that are all different sizes out of index cards and construction paper. Ask them how their diagonal knowledge will help to get perfect squares out of their paper. Students measure the sides (S) and diagonals (D) in cm and complete the chart (Blackline Master III - 22).

C. **Playing with Diagonals** Have students explore what happens to the diagonals when they rotate or enlarge polygons. Use computer applet: http://www.mste.uiuc.edu/m2t2/geometry/diags.html

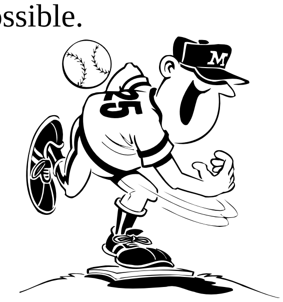
c.) Parallelism and perpendicularity of sides and diagonals.

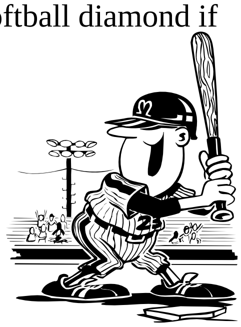
Notes and textbook references

A. Go Fly a Kite! Have students design and make a kite in the shape of any quadrilateral out of any appropriate material (tissue paper, construction paper, poster-board, etc). Students will need to use rolled up newspaper for the diagonals on their kites.

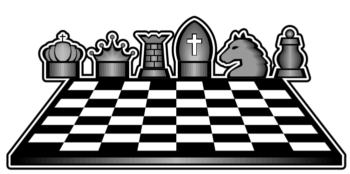
Note: Be sure to ask students questions to ensure they are considering properties learned about polygons and diagonals while constructing their kites. For example, consider asking, "How does the design of the kite affect the diagonals?" Kites may be flown if time allows.

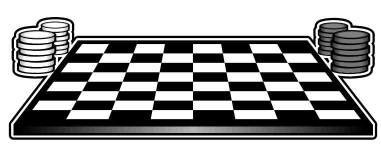
B. It's A Homerun! Baseball and softball are popular sports in our country. The boundary of the baseball diamond is a square, as you can see from the diagram. When the maintenance crew lays out a softball or baseball diamond, they carefully measure the diagonals to make certain the diamond is really a square. Without measuring, how can you be certain the diamond is laid out properly? What is the relationship between the diagonals? Test hypothesis on a real baseball or softball diamond if possible.





Note: This activity works with any situation that requires a perfect square.

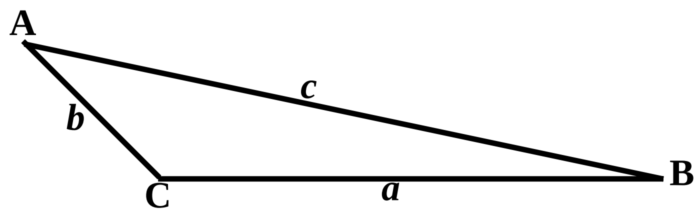




C. Attribute Game Divide students (4 to 6) into pairs of teams. Deal out all the attribute cards (Blackline Masters III - 23). One team makes a quadrilateral on a geoboard. Teams take turns playing a card that describes the quadrilateral on the geoboard. The last team to play a card wins the round and takes one card from the other team's deck. Play until one team has all the cards or until time is up. To make the game easier remove some cards from the deck. To make it more challenging, have students create their own decks.

Notes and textbook references

D. Have students label the vertices of a triangle [A, B, C]. Next have them label the sides opposite the vertices with the corresponding lower case letters [*a*, *b*, *c*]. Students then measure the angles and sides. Notice any interesting relationships? Students should observe that the largest angle is opposite the largest side, and the smallest angle is opposite the smallest side. See Blackline Masters II - 5 through II - 10.



Will this be true for other polygons?

E. Have students trace and measure the paths of the diagonals on the polygons (Blackline Masters II - 8 through II - 10). How do the lengths of diagonals compare with the lengths of the sides?