The learner will understand and use graphs and data analysis.

4

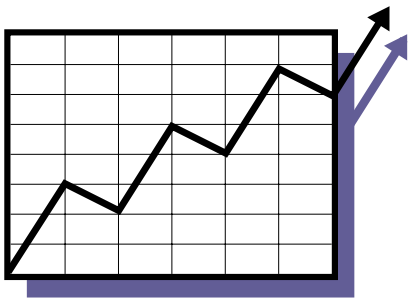
4.01 Collect, organize, analyze, and display data (including stem-and-leaf plots) to solve problems.

Notes and textbook references

A. Collect data using a variety of quick collection activities involving timing things to do in one minute. Complete the collect, organize, analyze, and display cycle. Example- How many X's can you draw in one minute with your right hand? Your left hand? Organize and display the data on an appropriate graph. Analyze factors that may have contributed to the discrepancies in the data or any outliers you may have. Why were some people able to draw more X's with their left hand? Would this data be appropriate for a stem plot? A line plot? A line graph?

The step of collecting data is often not experienced by students since many tasks already provide the data. Students need opportunities to work throughout the entire data collection process; pose a question, collect the data, analyze and interpret the data, and display it. Allowing students to collect data and make the different representations will increase their understanding of using data to solve problems.

B. Hold a class discussion on data collection and the importance of controlling all variables. Ask the students what they would need to do to help define the question before collecting data for the number of times a fifth grader can jump rope. For example: Can I jump as long as I want, anyway that I want, should someone else turn the rope, should I turn the rope, etc? Would we get an accurate answer to our question if some people skipped and others jumped with their feet together or if students were allowed to count their own jumps? Does everyone need to use the same type rope? It is important that students understand all factors that may lead them astray. Posing questions to ensure the data collected answers the question is more difficult than most students realize. Collect, analyze, and display the data.

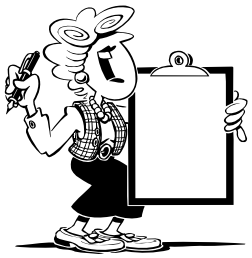


Notes and textbook references

C. Survey the class collecting data on the number of buttons on a student's clothing or the number of pencils in a student's desk. Display the data using an appropriate graph. Have students explain their graph and why they chose that representation.

D. Ask the students to collect graphs from newspapers and magazines. Working in groups of two to four, ask the students to study the graphs and make a list of questions which had to be answered to make the graphs. Have each group report to the class about their findings. Determine the questions that were alike/similar and make a list of decisions to be made before gathering data.

E. From the list below, have students determine how to graph the data given. Why are some types of graphs more appropriate than others?



- If the kindergarten students were measured each month to chart growth
- Number and different type of books in the library
- How students spend their allowance
- Number of free throws successfully completed by two people
- How students get to school - car, bus, foot
- The attendance of each fifth grade class for one week - displayed on one graph
- Student's age and shoe size (data on same graph)
- Student's height and weight (same graph)
- Daily grades on homework assignments for two weeks
- Favorite soft drink (from choice of three)
- Temperature in your city and temperature in another city for a two-week period
- Number of students in each grade compared to entire school

F. Give each pair of students data from real-life situations. Have the students generate a list of decisions to be made about arranging the data in graphic form. Ask the class to compile a list of different types of graphs and appropriate times to use each type of graph. Have the students determine which type of graph would fit their data.

Notes and textbook references

G. Assign class a data project by groups:

1. In groups of four, write an opinion question. For example, "Do you prefer Coke™, Pepsi™, or RC Cola™? Do you prefer Hardees™, McDonalds™, or Wendy's™? Which team will you root for - N. C. State, Carolina, or Duke?"

2. Develop a plan to gather data. Decide how many people to survey. What would be an appropriate sample?

3. After gathering data, organize it and decide how to represent the information in graphic form.

4. Create the graph.

5. Present graphs to the class and explain to the class why you chose that type of data display.

H. Have students collect data by hopping on their right foot for 45 seconds, while a partner counts and records the number of hops. Then, partners change places and repeat the same procedure. After gathering the data, students list the decisions that the class needs to make to organize the data into an appropriate display. What have you learned?

I. Continue the activity explained in 4.01 H . Once students have brainstormed a list of decisions that need to be made when representing data, they need many opportunities to gather, organize, display and interpret data. Spend time discussing how to be systematic about collecting data. If a group of students is working together to collect information, they need a system for knowing whether or not an individual has already been included in a data sample. Once a sample group is chosen, they need a system for making sure that everyone in the group is included. This might be a good time to *introduce* the idea of sampling and making generalizations about larger populations based on a sample. How does one determine whether or not a sample is representative of a total population?

If the questions that students brainstormed while working on 4.01 H need expanding in order to include a wider variety of content areas, this would be an appropriate time to extend the list of questions.

Notes and textbook references

Fifth graders might be interested in comparing themselves to some AMERICAN AVERAGES. Here are some interesting data from a book by the same name written by Mike Feinsilber and William B. Mead.

The average person:
** blinks 25 times a minute*
** laughs 15 times a day*
** makes 1,029 phone calls a year*
** reads 16 books a year*
** eats 5 times a year at Kentucky Fried Chicken*
** eats 92 hotdogs a year*
** eats 4 pounds of food a day*
** drinks about 1 bottle of soda a day (12oz.)*
** watches 3 hours of TV on an average weekday if 18 or older*
** watches 25,000 TV commercials a year if under 18*
** spends a day a year in a federal park*

The Average American Household:
** has the TV on 45.5 hours a week*
** washes 8 loads of clothes a week*
** receives 13 letters& bills, 2 magazines, & 4 ads a week*
** has 3 family members*

How are all <u>these</u> data gathered and analyzed?

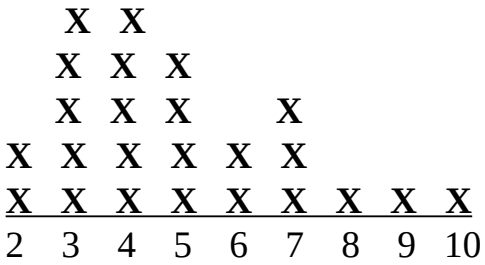
J. Research to find the population of five major cities in Canada, the United States, and Latin American countries. Have students organize the information and determine how to display data. Then, have students present information to another fifth grade class.

K. Is one paper towel better than another? What are the criteria for "better?" Is it strength? Is it the ability to absorb the most liquid? In working groups, choose a factor and design an experiment to gather data. Different groups can examine different factors and then share their data in class presentations.

Notes and textbook references

L. To help students understand the importance of clear titles and accurate labeling of their graphs, display the following and ask students to write sentences interpreting the data:

CARS



PEOPLE

After students try to interpret the graph, talk about why these labels are not clear enough. Does the graph describe the number of passengers who can ride in your car? Does it describe the number of cars people have brought over the past 10 years? The number of cars in your family? How people rated a given new car?

M. Have students bring to class a variety of food package labels to obtain nutritional data. Students are to collect data from the labels pertaining to calories per serving and fat grams per serving. Fill in the chart (Blackline Master IV - 1) from the data collected. Students can determine the amount of fat calories in a serving of each food.

Procedure: Multiply the number of fat grams per serving by 9 (calories per 1 gram of fat). Students can analyze the data to generalize which foods are healthier choices in their diet. Categorize foods by the food pyramid groups.

N. Have students watch television for specific hours and specific stations. Graph the number of commercials per hour. Compare Saturday mornings from 10-11 AM with Tuesday evenings from 6-7 PM or Friday nights from 8-9 PM.

Notes and textbook references

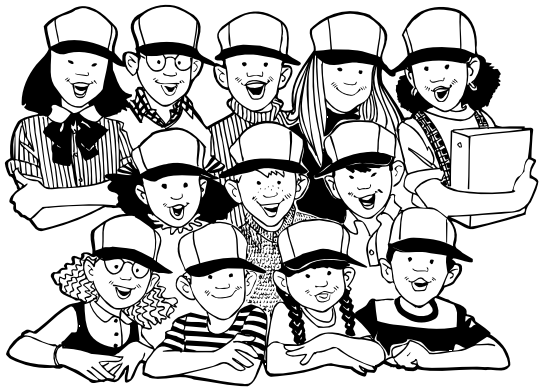
O. Before constructing graphs, students need to collect data to represent in a graph. Having them gather their own data is not only more engaging than taking already gathered information but also accomplishes objective 4.03.

Brainstorm a list of questions that students might like to answer about themselves. These questions might include the following:

- How long is a fifth grader's standing broad jump?
- How much taller is a fifth grader than a first grader?
- How many fifth graders fit into the principal's office at one time?
- What is a fifth grader's resting pulse?
- What is a fifth grader's exercise pulse?
- What is the typical family size of a fifth grader?
- How many sit ups can a fifth grader do in one minute?
- How many times can a fifth grader jump rope before missing?
- Try to encourage the inclusion of some questions that require the gathering of data over a period of time. For example, how many servings of fruit does a fifth grader eat in one week?

Choose one of these questions as a beginning point. Work as a whole class to plan a strategy for collecting the data in an organized fashion. Focus on questioning strategies in relationship to gathering information. For example, it is easier to gather and organize data when asking "Do you prefer hotdogs, hamburgers, or pizza?" than when asking "What is your favorite lunch?"

After the class has gathered a set of data, have small groups of students decide how they would display these data and be ready to justify their display. As each group explains and justifies its display, some issues or questions may emerge. Discuss the differences between the displays and what kinds of decisions that need to be made when creating a display. For example, What kind of graph will be used (line plot, stem and leaf, bar, circle, etc.)? How will the graph be labeled? What value will be assigned to each symbol? This list might become a poster that stays up as a reminder or kind of self checklist as students continue to create more graphs.



Notes and textbook references

P. Have students create charts and graphs showing the data that they have organized from activity 4.01 F. On each graph, they could also write a sentence that summarizes the information, ask a question, or draw some kind of conclusion. For example, a group that has charted or graphed data related to how students get to school might write: "Most of the students in this class ride a car to school," or "What might be the effect if the school buses didn't run one day?", or "If the school buses didn't run one day, half of our class wouldn't come to school." It is more important to discuss cause and effect statements, like the last example here. Other factors may also need to be considered. For example, bus riders might walk or get a car ride with a friend if the buses didn't run. Cause and effect statements appear in the media daily. In order to behave like responsible citizens, students need to recognize and question such relationships. Students might be asked to find examples of this kind of reporting in the newspaper and bring them in for discussion.

Q. Additional opportunities for charting and graphing data might come from organizing collections of various kinds. Bags of bean mix, candy hearts, M&M's™, colored paper clips, etc. could be sorted and organized by color or type. This information could then be used to construct a chart or graph showing the number of each color, etc. Have students summarize results. Students might be asked to bring in their own collections for sorting.

R. Have students record the outside temperature each day at 11 a.m. Allow them to organize and display the data weekly as part of an on-going investigation. Do you notice any patterns in the weather? What could be a contributing factor - high pressure, low pressure, cold front? Then, have students write an article for their class newspaper telling what they discover. Also, ask students to record monthly electric bills and the average temperature.

S. Have students record the names and ages of people at their death from an almanac. Direct them to graph their data in a stem-and-leaf plot. Have a second group of students collect data from current newspaper obituaries and graph that information. What comments can you make about each set of data?

Notes and textbook references

T. Model for students how to quickly collect data using a stem-and-leaf plot. Then help them organize the data (order it) and make summarizing statements. For example, draw the start of two stem-and-leaf plots on the board or overhead. Have students estimate the total number of pages they have read (any books) during the past week. Record their data. If the first 12 students gave the following numbers as their estimates, your quick graph would look like the sample on the left: 18 ,6, 9, 4, 15, 21, 19, 37, 28, 24, 33, 15. The data, when organized, would look like the stem-and-leaf plot on the right.

0	6, 9, 4,
1	8, 5, 5, 9
2	1, 8, 4
3	7, 3
4	
5	

0	4, 6, 9
1	5, 5, 8, 9
2	1, 4, 8
3	3, 7
4	
5	

U. Have students keep an hour by hour log for a 24 hour period. Ask the class to determine categories for their data (sleep, school, eating, play, and so on). Students need to round their categories to the nearest half hour or hour (nearest half hour is more precise, but may be very difficult for some students). Have each child construct a circle graph. (See pages 80 - 85, a section about creating graphs.) Use these graphs to create a bulletin board and have students write about their day.

Notes and textbook references

V. **Stem-and-Leaf Plot**: (also called stem-plot) is an easy way to represent a set of data. A stem and leaf plot works best for data with a range of several decades, since the plot is most frequently organized by tens. For example, here is a collection of data showing arm lengths in centimeters:

91, 77, 81, 78, 82, 64, 78, 69,

79, 77, 71, 71, 80, 54, 65

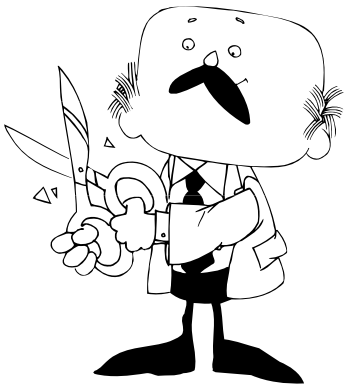
To make a stem and leaf plot of this set of data, divide each value into tens and units. The tens become the "stem" of the plot, and the units are the "leaves":

5	*4*
6	*4 ,5, 9*
7	*1, 1, 7, 7, 8, 8, 9*
8	*0, 1, 2*
9	*1*

In this plot, the first line shows one data point in the fifties: 54. The second line shows three data points in the sixties: 64, 65, 69.

Notice that the leaves in each decade are arranged in order. It is now easy to see many things. The range is the difference between 91 and 54, or 37. These data create an interesting pattern with a cluster in the seventies , and a form of a "bell curve."

W. Ask the students to bring in charts and graphs from newspapers and magazines (*USA Today, Newsweek, Science Digest*).
In groups, ask the students to review the charts/graphs and look for patterns and trends. Have the groups report their findings to the class.



Notes and textbook references

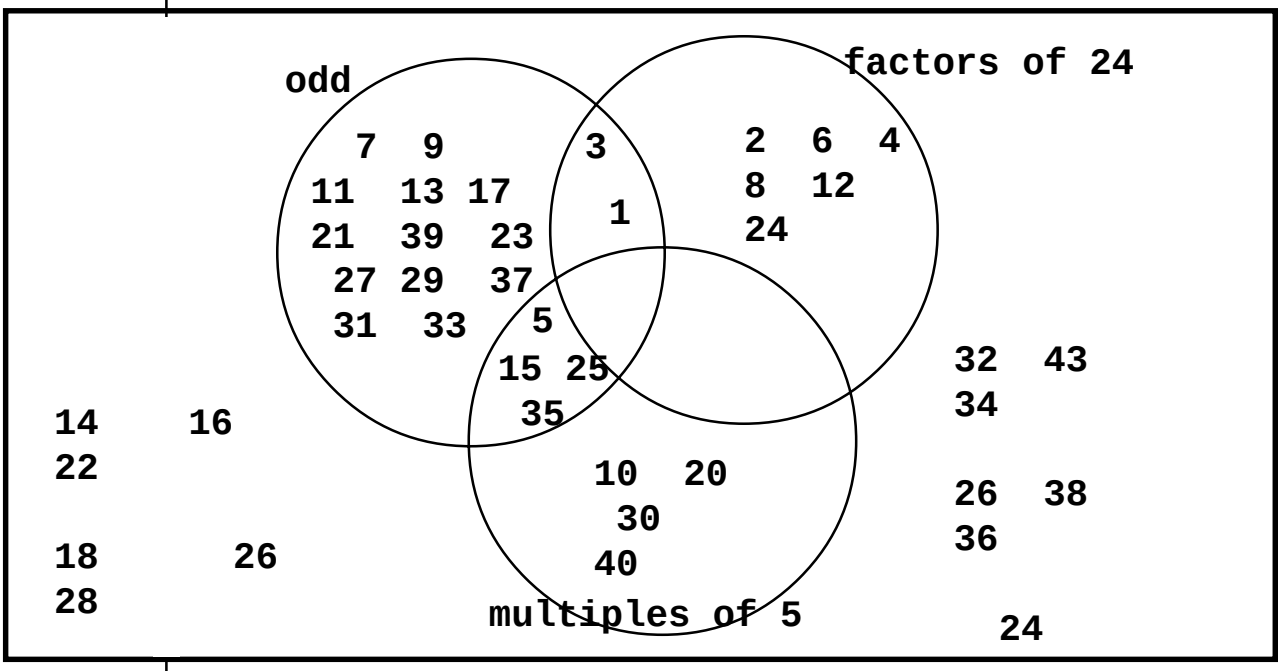
X. Work with a partner. Roll a pair of dice. Using the attribute chart label one circle on the Venn diagram with the attribute listed below the sum of the dice. For example, if you roll **3** your attribute is **odd**, **7** attribute - **factors of 24**, and **8** attribute - **multiples of 5**. See the chart below for all the categories.
This activity should be repeated several times. See Blackline Master IV - 2.

Attributes

2	3	4	5
even	**odd**	**one digit is 3**	**multiples of 7**

6	7	8
factors of 12	**factors of 24**	**multiples of 5**

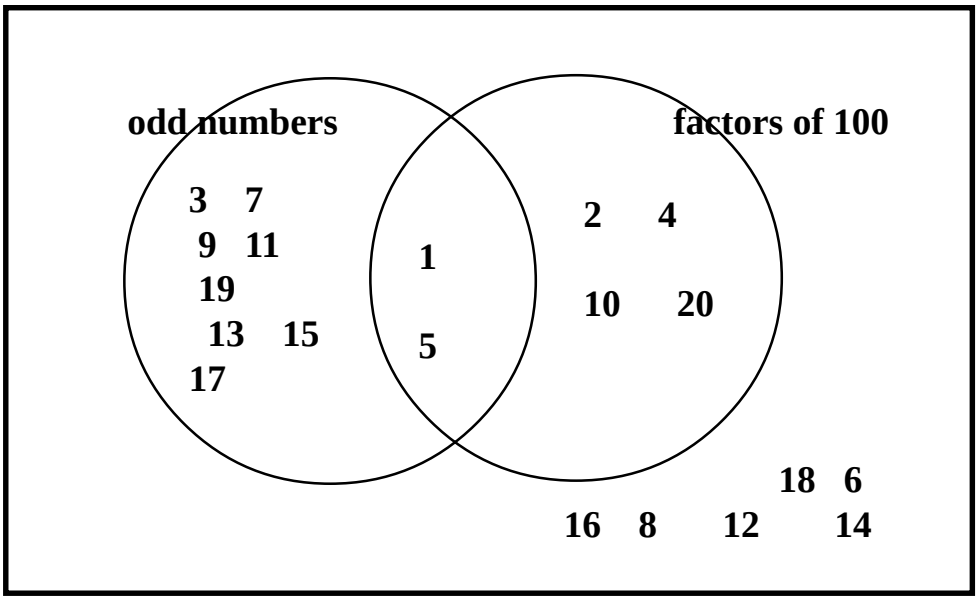
9	10	11	12
one digit is 5	**prime**	**multiples of 3**	**factors of 40**



Notes and textbook references

Y. Roll a die twice to get two of these attributes. If you get the same number on the second roll, keep rolling until you get a number different from the first. See Blackline Master IV - 3. Use the diagram to sort the numbers 1 to 20 according to the two attributes you rolled. For example, if you rolled 1 (**odd numbers**) and 4 (**factors of 100**), your diagram would look like the diagram below. Repeat the above steps two more times to make two different sortings of the numbers 1 to 20.
Extension: Have students work in pairs and assign the same tasks with the numbers 20 to 50, 40 to 70, 60 to 90 or 70 to 100.

Attributes		
1 **odd**	**2** **even**	**3** **factors of 25**
4 **factors of 100**	**5** **multiples of 4**	**6** **composites**



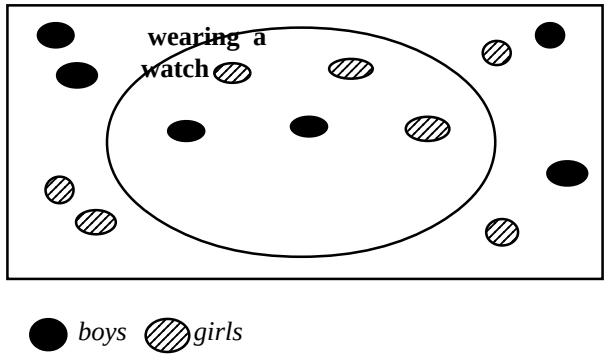
Notes and textbook references

Z. Venn Diagrams provide many opportunities for organizing complex sets of information. The relationships among ideas, animals, and objects can be graphically displayed. A Venn diagram can be used to show characteristics that are shared and/or not shared. The use of Venn diagrams can be personalized in classroom activities.

Begin with one idea, for example wearing a watch. Draw a circle on the chalkboard or a large piece of paper. Label this circle "wearing a watch." Explain that students who are wearing a watch will sign in the circle and those not wearing a watch will sign outside the circle.

Students begin working with Venn diagrams in 1st grade.

After students have responded to the Venn, ask a few simple questions about the data. How many students responded to the Venn? What fraction or percent of those who responded were wearing a watch? Suggest that it might be interesting to find out whether more boys or girls wear watches. Demonstrate how to draw a Venn Diagram that would generate this information. There are several ways to accomplish this. One is to have girls sign in one color and boys with another.



Another way to gather these data would be to draw two separate circles, one for boys and a second for girls. Then a third circle is drawn which overlaps both of these. This diagram creates four areas: A= boys not wearing watches, B= boys wearing watches, C= girls wearing watches, D= girls not wearing watches. Of course, if circles are used, there are some areas in which nothing really belongs, unless something other than girls and boys (male or female) wear watches. This can be "corrected" by using rectangles instead of circles. Venn Diagrams can be drawn with shapes other than circles, although circles are traditional.

A third way to draw this Venn would be to go back to the single circle and draw a horizontal line separating it and the area around it. Then label the top half with girls and the bottom with boys (or draw a vertical line and use left and right labels). Continue posting Venn Diagrams for student response. Have students brainstorm ideas for Venn diagrams. If a magnetic board is available, each student could have a small magnet with his/her name on it to place on each Venn. There may be times when a

sensitive question is asked. Students could respond in an anonymous manner by marking "X" marks or placing a colored circle label in the appropriate place.

Provide many opportunities for students to label and organize information on increasingly more complex Venn Diagrams. After exploring two circle diagrams, move on to three. How many different ways can three circles be drawn? Provide lists of categories for students to diagram, for example, factors of 18 and factors of 24; birds, frogs, plants, and living things; math papers, playing soccer, and homework; people who travel, pilots, people who fly in airplanes, and people; sandals, boots, and shoes, etc. Reverse the process also by supplying Venn Diagrams and asking students to provide labels.

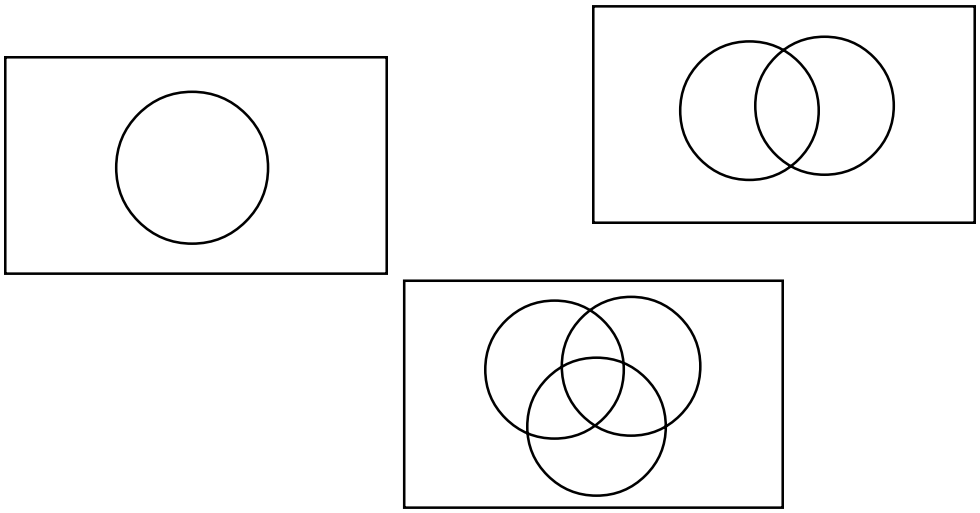
Venn Diagrams also provide a good strategy for solving problems. Here is a sample: One day a zoologist spotted 30 animals in the jungle. Seventeen had fur, 21 had tails, and 2 had neither. Of the animals she saw, how many had both fur and a tail? By setting up four regions on a Venn and then moving markers around into the regions until all the information fits, one can solve this problem. The regions are: A= animals with fur and no tail, B= animals with a tail and no fur, C= animals with fur and a tail, D= animals with neither fur nor a tail. If 2 animals had neither, then 30 - 2 = 28 animals that belong in regions A, B, and C. How can 28 be grouped into these regions so that a total of 17 are in A and C together and 21 are in B and C together?

GET IT TOGETHER by EQUALS and Cooperative Problem Solving with Calculators by Creative Publications both contain more problems of this type.

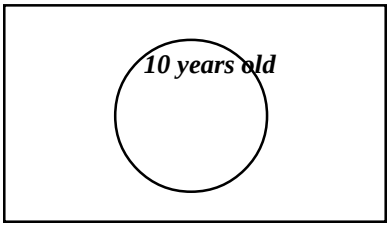
Notes and textbook references

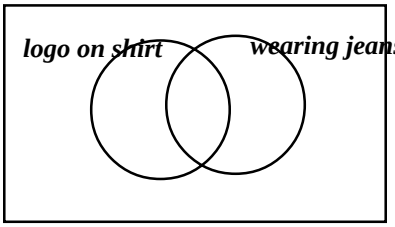
Notes and textbook references

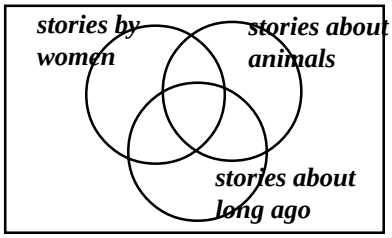
AA. Keep a set of laminated Venn diagram posters like these:



On a weekly basis, change the names of the circles and have the students put their names or the item in the appropriate sets. At the end of the week, discuss what the diagrams show. Examples:







4.02 Compare and contrast different representations of the same data; discuss the effectiveness of each representation.

Notes and textbook references

A. Use the class survey sheet from week one of the Week-by-Week Essentials to gather data on each of your students. Allow students the opportunity to graph the results for one of the questions using multiple representations. Note: Let the students spend time grappling with making these graphs. It is a great tool to help them realize which graphs are good for which kinds of data. Hold a class discussion on the types of graphs and what kinds of data are most effectively displayed on them. Discuss pros and cons for each graph.

B. Ask the students questions to assess their understanding of this objective. Example: If everyone in our class scores between 90-99 on the graphing test, would a stem-and-leaf be a good graph to display the data? (No, since all the data falls on the same leaf.) Which graph would work well?

C. Each student gets a box of raisins. Estimate, then count the number of raisins in each box. Put the results of the number of raisins in each box on a line plot. Use another brand of raisins and repeat the process except place the data on a stem-and-leaf graph. Use a third brand of raisins and repeat the process but construct a bar graph. Discuss the three types of graphs and draw some conclusions about the graphs.

D. **Double Bar Graphs vs. Double Pie Charts** - (Blackline Master IV - 4 through IV - 5) Students interpret information from a double bar graph and use it to create two pie charts. As the students answer the associated questions, they should find that some information is more easy to obtain from one form of the graph than the other.

E. **Bar Graph vs. Line Graph** - (Blackline Master IV - 6) Students interpret data from a bar graph and use it to create a line graph. As the students answer questions associated with the data, they should find that some information is easier to obtain from one form of the data than the other.

Notes and textbook references

4.03 Solve problems with data from a single set or multiple sets of data using median, range, and mode.

It is critical that students understand the difference with categorical and numerical data before attempting to find the median, range and mode of a data set. Categorical data deals with categories or words. For example: data displaying favorite foods is categorical since the data is in word form; pizza, steak, enchiladas, etc. Numerical data deals with numbers. Since words cannot be subtracted, there can be no range. Categorical data can also not have a median because there is no middle number! Note that students may try to use the y-axis (count) to find the range and/or median. This misconception needs to be corrected immediately when it occurs.

A. Have students collect data on their heights. Divide students into groups of three or four. Each group makes an index card set (one card for each person's height). Color-code the cards; blue markers for boys' height cards and red markers for the girls' cards. Each group works together to display the cards to determine the range, mode, and median for the boys' height data set and girls' height data set. Each group needs to share its strategies for determining the measures of central tendency and the range. After a class discussion, ask students how the measures and ranges would change if a new student (indicate a girl or boy) walked into our class. Allow the students to determine what the new student's height might be to change the mode of our data? The range? The median? For example, the teacher may ask the following questions:

- How tall would the student need to be to change the range?
- What height might the student be if the median changed?
- Name a possible height for the new student that would change the current mode. Would any other heights work?
- Have students share their solutions.

B. Using a county or city map, ask the students to develop multi-step problems related to measurement, graphing, and statistics. Example: Compare the number of intersections of streets (roads) in two one-mile squares on the map.

C. Pretend that you are a stock broker. One of your clients would like you to invest her money by selecting a stock from the New York Stock Exchange. Record the progress of the stock that you selected for ten trading days. Graph the progress of your client's stock. Then, write a report to your client informing her of the stock's progress including the range, median, and mode. Was this a good investment? Explain why or why not.

Notes and textbook references

D. Survey the class to determine the number of members in each student's family. Students need to define "family" as a class discussion. Identify the range, median and mode. Graph data.

E. Using a data chart from your school system or your social studies book, ask the students to develop multi-step problems related to the data. Have them work in pairs or groups. Trade problems among the groups until the students have worked on all the problems.

F. Ask students questions and have them determine whether the question would collect categorical or numerical data. Example: Numerical - "How many pets do students in class have?" Categorical - "What kinds of pets do students in our class have?" Chart the class results for types of numerical and categorical questions. Be sure to identify and discuss the measures of central tendency.

G. Collect data for one of each question type, numerical and categorical. Have students make bar graphs of the data and determine the mode, range, and median, when appropriate. Be sure students do not confuse the range of y-axis with the data points.

H. Have students gather sport statistics (i.e., baseball cards). With these statistics, rank data from least to greatest and determine range, median, and mode. Sets of data can be plotted and compared using a variety of graphic organizers which include line graphs to study trends, bar graphs, and stem-and-leaf plots.

I. Ask the students to bring in information from newspaper and catalogue advertisements about buying a television. Include the price, size, length of warranty and other pertinent information. Arrange the data in various ways so that the students (in groups) can make generalizations. Establish criteria to help determine the best buy. Discuss measures of central tendency. Predict which television might be the best seller.

Notes and textbook references

Some thoughts on graphing . . .

We live in an information age. Rapidly expanding bodies of knowledge combined with increased uses of technology clearly indicate that we as adults and the children currently enrolled in elementary grades will need to be able to evaluate and use vast amounts of data in personal and job-related decisions.

Skills in gathering, organizing, displaying, and interpreting data are important for students within all content areas. Graphing activities incorporate knowledge and skills from a variety of mathematical topics and integrate geometric ideas with computational skills, and classification tasks with numeration understandings.

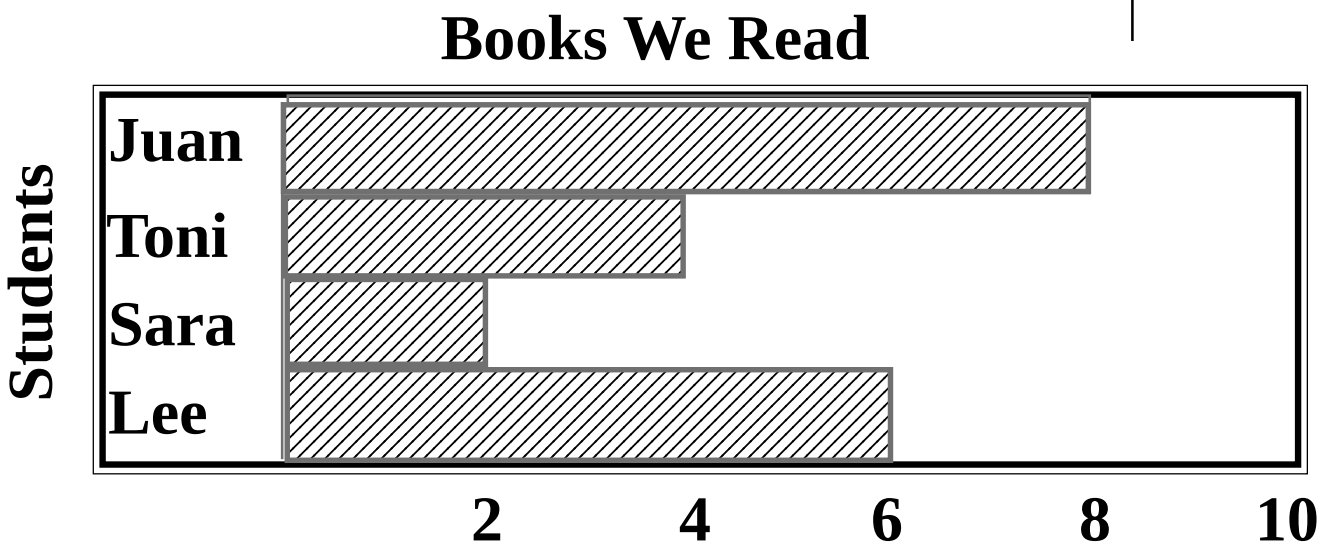
Graphs provide a means of communicating and classifying data. They allow for the comparison of data and display mathematical relationships that often cannot easily be recognized in numerical form. The traditional forms of graphs are picture graphs, bar graphs, line graphs, and circle graphs. New plotting techniques include line plots, stem-and-leaf plots, and box plots.

The following discussion is paraphrased and condensed by permission from the National Council of Teachers of Mathematics from Developing Graph Comprehension, *pages 1-9. Information on newer techniques is for the teacher's benefit, since most applications of these techniques are more appropriate for upper elementary and middle grades rather than fifth grade.*

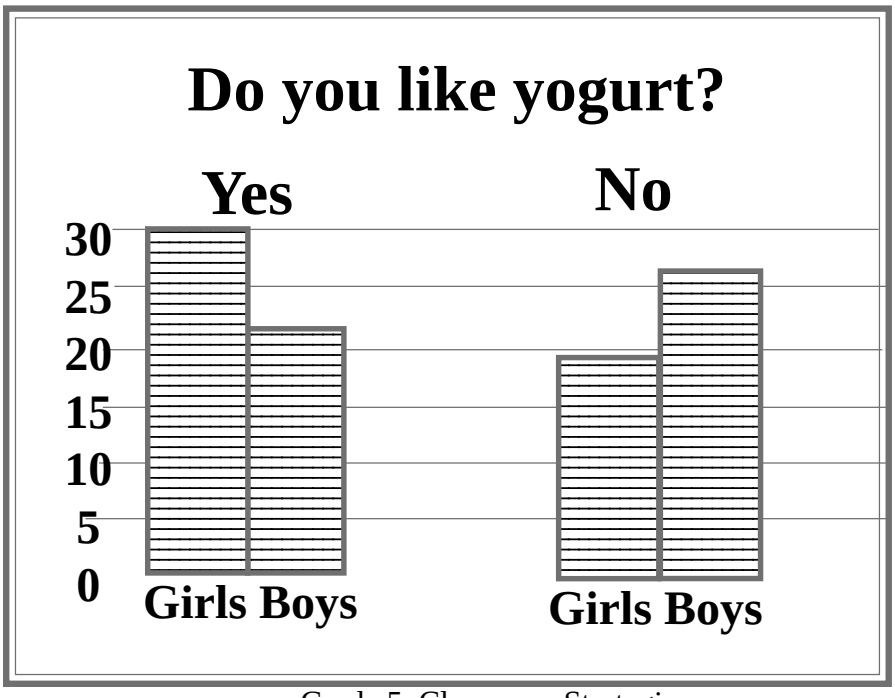
Traditional Graph Forms

Notes and textbook references

Bar graphs. *Used horizontally or vertically, bar graphs (also called bar charts) compare discrete quantities expressed by rectangular bars of uniform width. The heights (or lengths) are proportional to the quantities they represent. The bars are constructed within perpendicular axes that intersect at a common reference point, usually zero. The axes are labeled.*



Multiple or double bar graphs are used to compare discrete stratified date (i.e., data collected from particular groups). For example, when asking children to vote for their favorite pets, colors, or favorite games to play, organize the results according to boys' responses and girls' responses.



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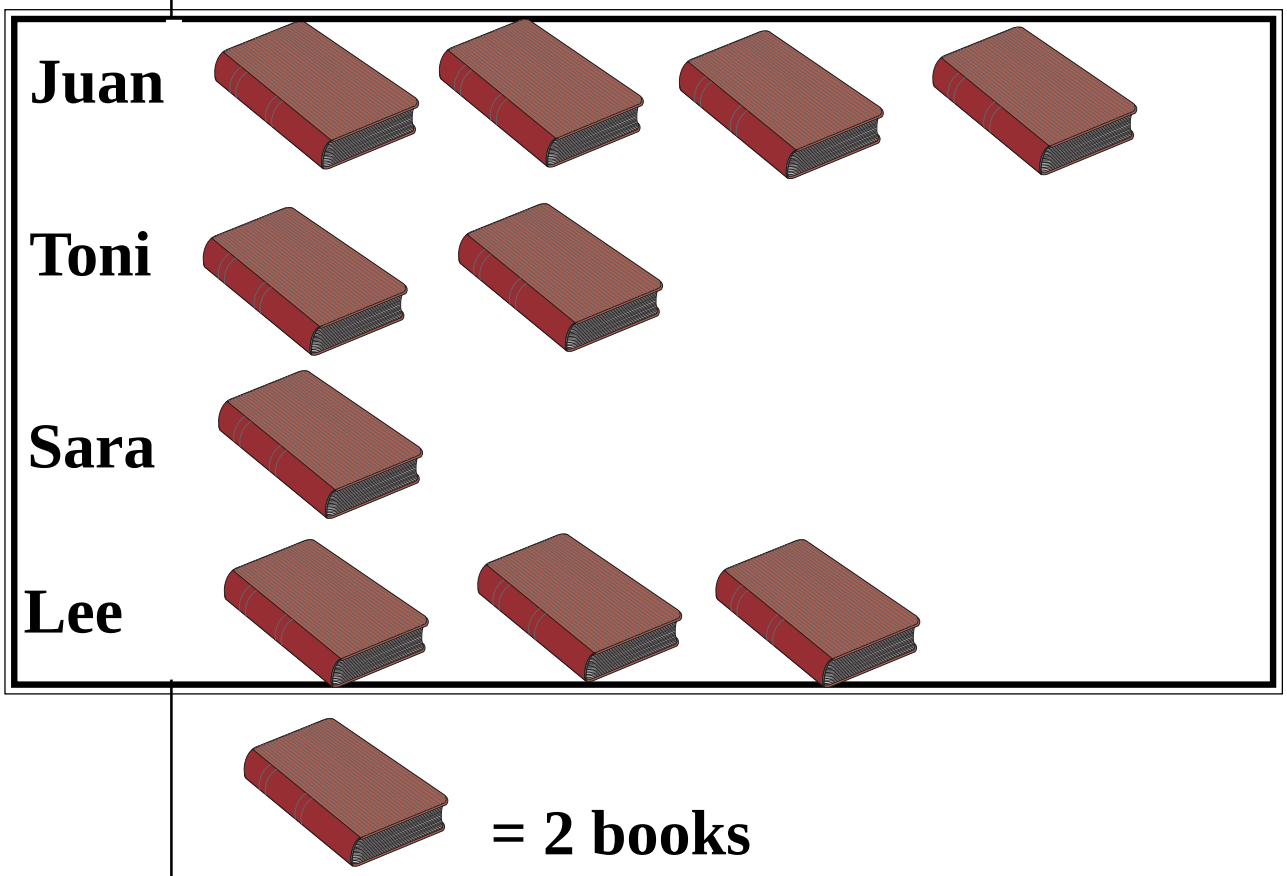
Books We Read



Picture graphs. *Picture graphs (pictographs) use the pictures to depict quantities of objects or people with respect to labeled axes. They are used when the data are discrete (i.e., noncontinuous). The symbols (ideographs) need to be the same size and shape. These symbols may represent real objects (e.g., a stick figure to represent a person or a carton to represent milk drunk by students) or they may take the form of something more abstract (e.g., a triangle or square).*

In picture graphs without a legend, the ideograph and the item it represents are in a one-to-one correspondence. When a legend is used, the ratio of each ideograph to the number of objects it represents must be taken into consideration when interpreting the graph. Fractional parts of ideographs (e.g., one-half of a picture) may cause some difficulties for children. Data presented in picture graphs are usually appropriate for bar graphs. Converting picture graphs to bar graphs is one way to help children move from semiconcrete representations of data to more abstract forms.

Line graphs. *A line graph (broken-line graph) is used to compare continuous data. Points are plotted within perpendicular axes to represent change over a period of time or any linear functional relationship. The labeled axes intersect at a common point, usually zero. The units of division on each axis are equally spaced, and the graphed points are connected by straight or broken lines. When children keep a record over a period of time of their own height or weight, of the daily average temperature, and so on, line graphs are appropriate displays.*

Multiple line graphs are used to compare two or more sets of continuous data: for example, to compare the heights or weights of two children over a period of time (e.g., four months or one year), or the heights of two (or more) plants over a period of time (e.g., one to two months after planting seeds).

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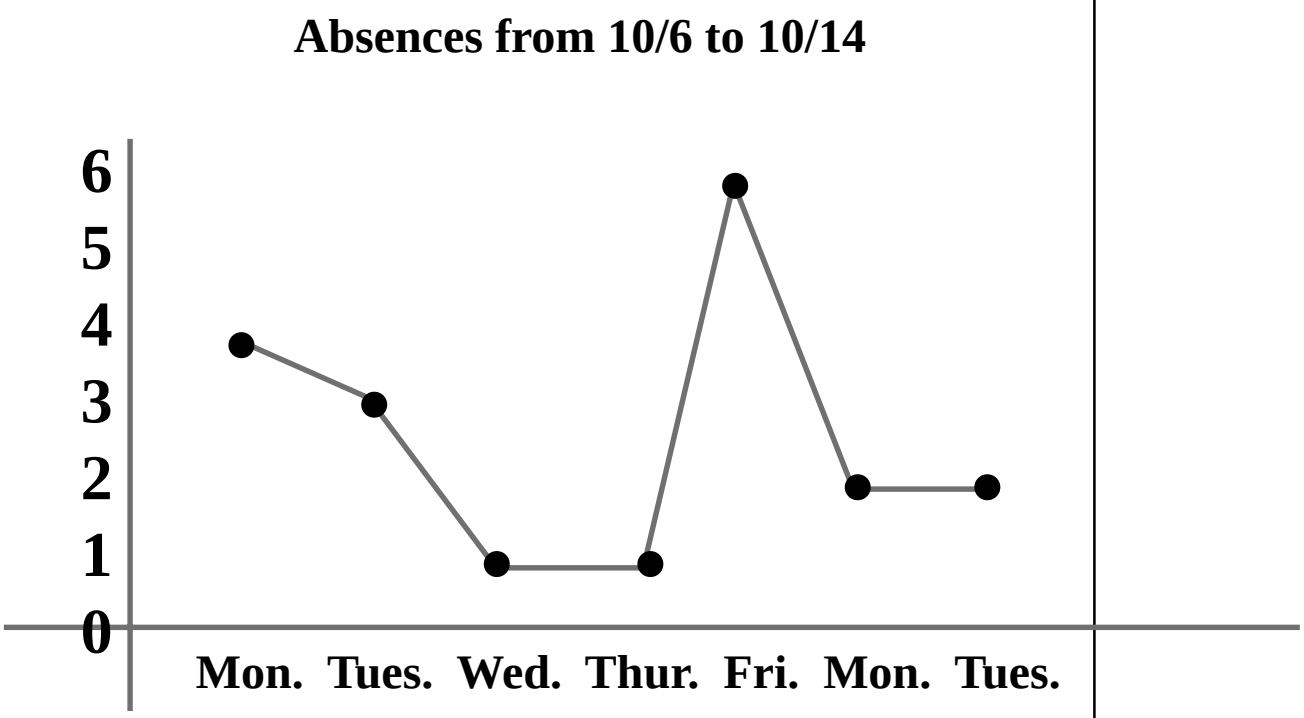

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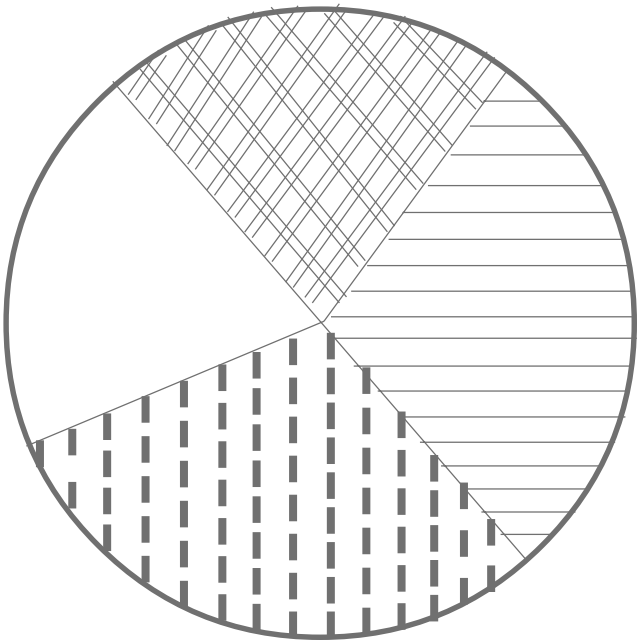
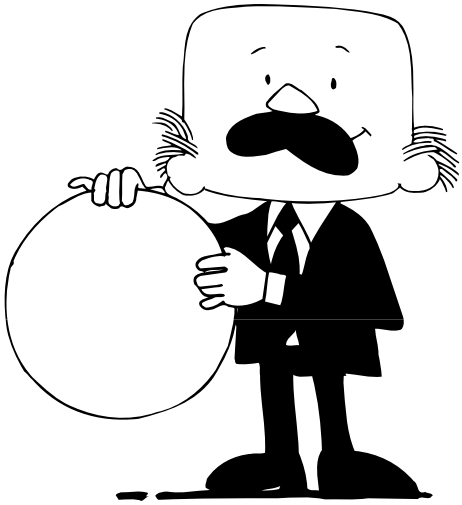
Circle graphs. *The area of the circle graph (pie graph, pie chart, area graph) is divided into sections by lines emanating from the center of the circle. Circle graphs are appropriate when children have an understanding of fractions; they provide children with a means of displaying the relationship of parts to whole.*





Children may create circle graphs informally before they are able to measure angles and figure proportions. For example, counters representing the total units are evenly spaced around the circle. When the divisions occur, a radius is drawn to divide the circle into appropriate parts. A second informal method is to mark units on a strip and then loop to form a circle, drawing radii as appropriate.

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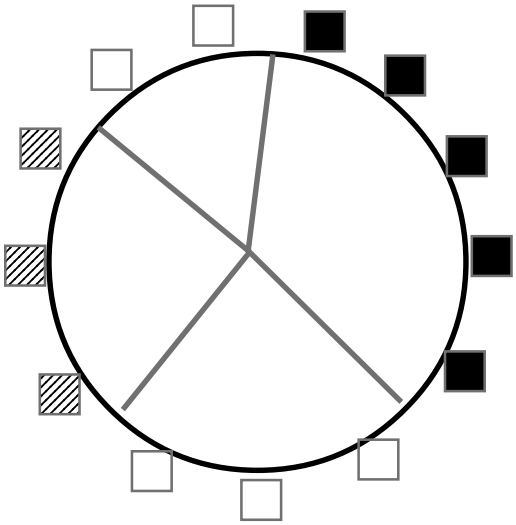

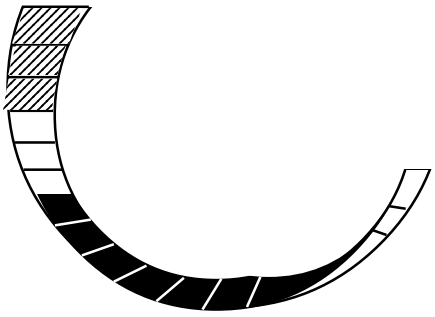

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