

The learner will demonstrate an understanding of mathematical relationships.

5

5.01 Identify, describe, and generalize relationships in which:

Notes and textbook references

a) Quantities change proportionally.

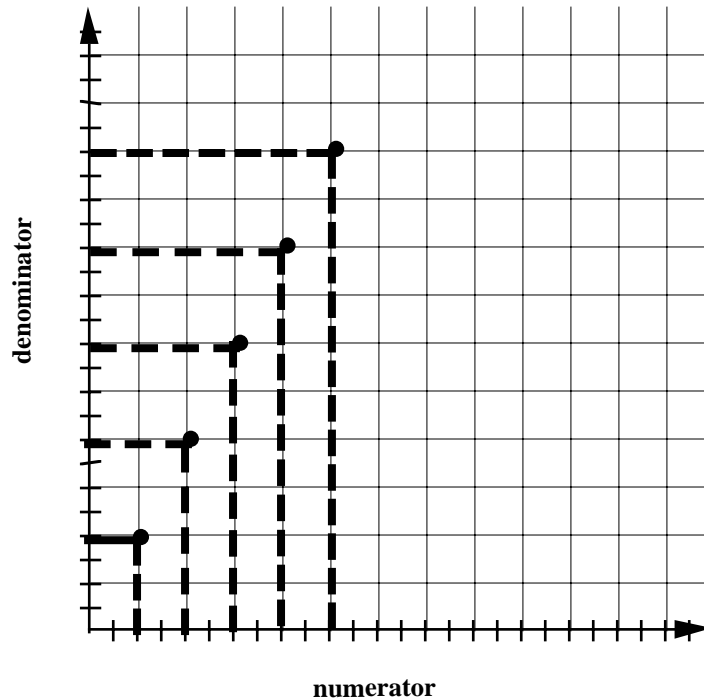
A. Ask students to collect recipes from magazines, newspapers or food boxes. Students work in pairs to change the number of servings needed. Different pairs then exchange recipes and find the new quantity of ingredients needed.

B. Science explorations offer an opportunity to examine rates and change in quantities. Students can compare the growth of seedlings, distance and its effect on the force of magnetism, weights on the speed of car designs, and other variables in science investigations

C. Anno's Math Games II by Mitsumasa Anno also offers an opportunity for students to explore function machines and the relationship between inputs and outputs.

D. Give students a set of equivalent fractions to plot on a coordinate grid (Blackline Master V - 5) with the numerator on one axis and the denominator of the other axis. Draw horizontal and vertical lines to each plotted point to form a rectangle (see example). Ask students what they notice about the rectangles (they are similar) and the fractions (they are equivalent).

Polygons are similar when all corresponding angles are equal and the ratios of corresponding sides are equal.



$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{4}{8} \quad \frac{5}{10}$$

Have student continue to explore:

- Do all equivalent fractions make similar rectangles?
- What happens when you use fractions that are not equivalent?
- What do you notice if you connect the plotted points? (It forms a straight line if the fractions are equivalent).
- How do you know that equivalent fractions are proportional? (The relationship between the numerator and the denominator is the same).

E. Cartooning Students will redraw a picture proportional to a graphic. See Blackline Master V - 6.

Materials: Blackline and paper cut in the shape of a large square; colored pencils

Procedure:

Ask students to cut out the small graphic on the Blackline

Students next fold the graphic in half twice. Students should open the graphic and draw lines along the folds. If done correctly, there should be a 4 by 4 array (16 squares).

Student should now fold the large square twice. The paper should be opened and lines drawn on the folds. There will also be a 4 by 4 array (16 squares).

Ask students to redraw each small square from the graphic onto the appropriate square on the large sheet of paper. When done, the new picture will be proportional to the graphic originally used.

Colored pencils may be used to finish the picture.

F. Exploring Proportion with Square Tiles

Materials: Color tiles, graph paper, colored pencils, and plain, white paper.

Procedure:

1. Begin the activity by instructing each student to use the square tiles to build a rectangle with an area of 6 units.
2. Instruct them to build a second rectangle that is proportional to the first. Observe the area and dimensions of the students' figures and have them share their figure.
3. Begin a discussion to improve students' understanding of proportionality. Students' should leave with the following understanding rather than a rule to be memorized!

To make a proportional figure multiply or divide all dimensions by the same number.

b) Change in one quantity relates to change in a second quantity.

A. Pose a problem such as this: Meghan has a drink stand. She sells one and a half gallons of lemonade and one fourth of a pint of orange juice each day. She has 10 gallons of lemonade and 2 pints of orange juice prepared. Which of the two drinks will be gone first?

Encourage the students to construct tables that highlight the changes in the quantities.

Number of Days	Gallons of Lemonade Sold	Pints of OJ Sold
1	$1 \frac{1}{2}$	$\frac{1}{4}$
2	3	$\frac{1}{2}$
.	.	.
.	.	.

B. Use charts like the one below to solve problems such as the following:

- How many calories are in a dozen doughnuts if each doughnut has 95 calories?
- How many half dollars are in \$12.00?
- You owe your mother \$16. 50. You agree to pay \$1.50 until the debt is paid. How many weeks will it take to pay the debt?

Donuts	Calories
1	95
2	190
3	?
4	?
.	
.	
?	950

NCTM Navigating Through Algebra Grades 3-5 is an excellent resource for additional activities. ISBN 0-87353-500-6

C. Use function machines (In / Out Machines) to explore changes in quantities. Ask students to describe the relationships between the numbers horizontally as well as vertically. After working with machines like those below, allow students to make their own machines to exchange with others. Encourage the use of more than one operation in a rule. Discuss strategies for using the rule when the out number is given (using the reverse operation).

IN	OUT
4	100
1	25
8	200
2	?
?	500

Rule: $IN \times 25$

IN	OUT
2	1
3	3
4	5
6	?
?	17

Rule: $IN \times 2 - 3$

IN	OUT
80	20
50	50
25	75
?	0
1	99

Rule: $100 - IN$

D. The book Two of Everything by Ling Toy Hong is a Chinese folktale about a magical pot that doubles whatever is put into it. Read the book to the students and then ask them to create a function machine that would work like the pot. Encourage the use of symbols such as

$\square = IN$ and $\blacktriangle = OUT$ to show relationships between the quantities such as $\square + \square = \blacktriangle$ or $\blacktriangle = \square \times 2$. As an extension, students can then write their own folktales using other functions and create tables to match their tales.

Suggested Classroom Accommodations for Students with Specific Learning Disabilities

Cognitive Strategies	Behavior	Accommodations
Remembering	forgets order of steps	chart of steps displayed
Self-managing	cannot explain concept	self-questioning taught
Information gathering	does not understand on first listening	frequent summaries paraphrasing strategy
Organizing	cannot make visual representation	vocabulary recorded with both words and a visualization strategy
Analyzing	cannot locate errors	verbal rehearsal strategy
Problem solving	cannot shift strategies	demonstrate each problem using two strategies
Time managing	poor assignment completion	prioritize assignments; required time chart for increased awareness of time demands
Integrating	poor notes	note taking strategy organized by concepts, not textbook chapters
Generating	weak concept connecting	prediction strategies pattern awareness
Evaluating	poor test taking	alternate tests; frequent assessment; test taking strategies

Some Additional Accommodations

- | | |
|---|--|
| <ul style="list-style-type: none"> ◆ Modify original task to meet the needs of handicapped students. ◆ Provide taped material to listen to, rather than read. ◆ Emphasize higher use of objective test in contrast to subjective tests. ◆ Offer three choices instead of four in multiple-choice formats. | <ul style="list-style-type: none"> ◆ Provide highlighted text for student use. ◆ Provide large print materials. ◆ Increase allowable time for completion. ◆ Reduce weight of test importance. ◆ Change fill-in-the-blank to multiple-choice format. |
|---|--|

5.02 *Translate among symbolic, numeric, verbal, and pictorial representations of number relationships.*

Notes and textbook references

A. Prepare hundred boards for the students and one hundred board transparency. Place a chip on a starting number such as 47 and ending number such as 89. Ask students to come up to the overhead and use different colored markers to show a path from the beginning number to the ending number. What is the quickest path to the ending point. Introduce the use of arrows to record the path.

$$47 \downarrow \downarrow \downarrow \downarrow \rightarrow \rightarrow 89$$

Use arrows to indicate different movements on the board. Look for patterns in the movements. What does a diagonal move south to the right mean in terms of the change in quantity of the beginning number? What about a horizontal move to the left? Ask students to predict what ending number a series of arrow movements will land them on. As an extension, students can create their own arrow patterns and exchange them with a partner. See Blackline Masters V - 1 and V -2.

B. Draw a tic tac toe grid such as the one below. Put a number inside each grid space.

1	2	7
3	6	4
8	5	9

Have the students identify the symbol for each number (i.e. $7 = \sqcup$).

Show the students an equation such as :

$$\square + \sqcap = 11$$

Students should work in groups to replace the symbols with the numbers they represent and then solve the equation. After a few examples, ask students to create their own equations, which they will exchange with another group. As an extension, students may create their own grids.

Use the books *Mathematicians are People, Too: Stories from the Lives of Great Mathematicians* by Luetta and Wilbert Reimer, published by Dale Seymour, to give the children a bit of the history of mathematics and the people who have made significant contributions to what we know. Volume 2 has a story about Fibonacci. ISBN for volume 1 is 0-86651-509-7; for volume 2 is 0-86651-823-1.

C. Explore patterns with geometric design relating the term (position in the pattern and the number of segments used to create the design. Have students create the patterns with toothpicks. Make a corresponding chart and look for patterns. Encourage students to find several ways to describe the pattern. Ask questions such as : How many segments are in the 10th term? In the 14th term? Encourage students to generalize the relationship between the number of the term and the number of segments?



Have students create a geometric design with the triangular pattern blocks. Explore the number of sides in a one-triangle design, in a two-triangle design, and so on. Use a chart to organize their findings. What patterns can they observe?

What happens to the number of sides each time you add a triangle? What is the perimeter for a four-triangle design? A ten-triangle design? Continue to explore patterns with the other pattern blocks.

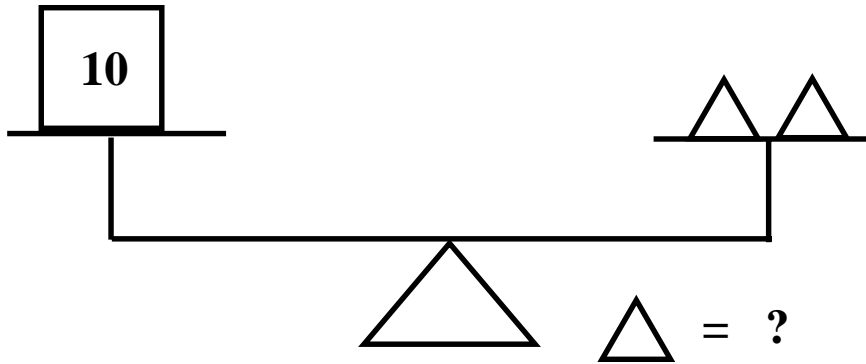


Explore the Fibonacci sequence with the students. Do research to find out about this amazing mathematician.

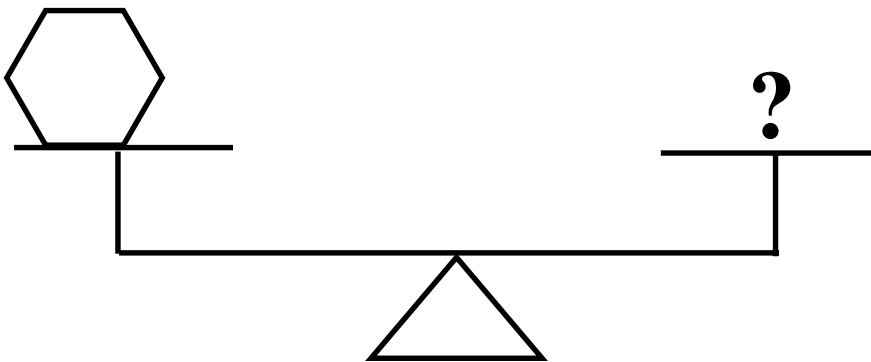
D. Nature makes great use of patterns. Have students bring in plants or leaves and investigate and describe growth patterns. Students might notice the number of petals on certain flowers or how leaves alternate on a plant stem or the number of points on a leaf, etc.

E. Prepare a hundred chart for each student and overhead copy for the teacher. Trace a rectangle somewhere on the chart. Add the numbers in the opposite corners. What do you notice? Try another rectangle. Do you see a pattern in the sums? Try a square. Compare the results. Are there other shapes that produce similar results? Extend the strategy by using a 200 or 300 chart. (See Blackline Masters I - 40 and I - 41)

F. Use balance problems to encourage the translation of symbolic to numeric representations as students examine number relationships. Use the balance template (See Blackline Master V - 4) Make a transparency of the template and use overhead pattern blocks to present balance problems like this:



Establish a value for the triangular block and continue the problems with this value, i.e...



Students then use the fractional values of the blocks to find the value of each side of the balances. Students can create their own balance problems to trade and solve.

5.03 *Verify mathematical relationships using:*

a) *Models, words, and numbers.*

A. Number Pan Balance Teacher uses e-tool applet from www.nctm.org as a class demonstration or sends students to this site (<http://illuminations.nctm.org>) in the computer lab.

Students explore the number pan balance by completing the tasks on this site. Teacher then asks the class to put 25 in the left pan balance and to write expressions that balance the 25. Example: 25 balances with $20 + 5$ ($25 = 20 + 5$). The teacher can encourage students to explore order of operations and number properties during this online investigation. After students have completed the number pan balance, they should move into the Shape Pan Balance Applet.

B. Two of Everything

Objectives

- Have students create and interpret rules
- Represent information numerically on a T-chart
- Describe first with words and the algebraically

Materials:

- Two of Everything by Lily Toy Hong
- Chart paper
- Vocabulary-Input value, output value, variable, T-chart, equation

Part 1

- Read and discuss the story Two of Everything by Lily Toy Hong.
- Draw a T-chart on the board as a way to keep track of what goes in and what comes out.

Ask:

- Suppose we put 5 coins in the bag, how many would come out? How do you know? Suppose we put in 4 coins, how many would come out? How do you know? What about 3 coins?
- Students should respond with two different ways to solve the problem.

input	output
1	2
2	4
3	6
4	8
5	10

- (Add input to input or the input times 2.)
- Reverse the question . What if the output is 4, what is the input value? How do you know? If there more that one way to solve this?
- Discuss patterns seen by student on the T-chart. Patterns seen may include the input goes up by one and output goes up by two, multiply the input by two and you get the output, or add the input to itself and you get the output.
- Examples (record on board as students share)

$1 + 1 = 2$	$1 \times 2 = 2$
$2 + 2 = 4$	$2 \times 2 = 4$
$3 + 3 = 6$	$3 \times 2 = 6$
$4 + 4 = 8$	$4 \times 2 = 8$

Extend pattern: What if we put in 10 coins how may will we get out? What if the input is 100, the output will be ____? How do you figure your answer? Suppose your output is 100, how many coins did you out in? How do you know? If there more than one way to figure this?

Generalize: Write a rule using symbols to explain the input/output.

- Look back at the patterns. Who can describe this pattern with a rule?

$1 + 1 = 2$
$2 + 2 = 4$
$3 + 3 = 6$
$4 + 4 = 8$
$5 + 5 = 10$

The output is equal to input plus input or input plus input equals output.

- How do you describe the following pattern?

$1 \times 2 = 2$
$2 \times 2 = 4$
$3 \times 2 = 6$
$4 \times 2 = 8$
$5 \times 2 = 10$

The output is equal to the input number times two.

Extend:

- Introduce symbols/variables and equations. Use a square for the input value and a triangle for the output value.

How would you write the rule output is equal to input plus input?

$$\square = \triangle + \triangle$$

Notes and textbook references

input	output
1	2
2	4
3	6
4	8
5	10
10	20
.	.
.	.
.	.
.	.
100	200

input	output
1	2
2	4
3	6
4	8
5	10
10	20
.	.
.	.
.	.
.	.
100	200

Now have students use the variables to write a mathematical sentence for the rule output equals input times two.

$$\square = \triangle \times 2$$

Part 2

Objective:

Apply previous knowledge to determine rule for input/output and to write the rule in an equation using variables.

Procedure:

- The teacher will create a second rule and follow the procedures listed below to help students determine the rule, write the rule, and then use variables to write the rules in an equation.

The teacher will give the information on the chart above and will ask if any one knows what the output value will be if the input is 2? If the response is incorrect then the teacher will complete the output for 2 and record the next input value.

Example: The student says the output is 4. The teacher says, “This is not correct. The correct output for 2 is 5. What is the output if the input is 3?” The student then has another clue to help them determine the rule. Continue this proves of giving the correct output if the response is incorrect. Then give the next input and ask for the output. Once a correct response is given continue with the pattern until you have received three or more correct responses before you ask for a rule.

Example: The student says if the input is 3 then the output is 7. The teacher will respond with “Correct!” but will not ask for a rule. Instead the teacher will continue to give a new input and request the output until receiving three to four more correct responses. Then you are ready to ask for a rule. Always begin the rule with “The output numbers equals . . .”

Example: The output number equals the input value times 2 plus 1. After writing the rule in words then write the rule as an equation using variables.

$$\begin{aligned}\text{Input} &= N \\ \text{Output} &= M \\ M &= N \times 2 + 1\end{aligned}$$

- Follow this procedure one more time with a teacher-invented input/output. Then ask the student to create a rule and T-chart to share with the class. Follow the same procedure.

Part 3

Assessment

Objective:

- This day is spent with pairs of students creating their own T-chart based on a rule they create or select from a list.

Procedure:

- First have the class brainstorm (record on a chart) ten possible rules.

Examples:

Add by 2

Double and subtract 5

Multiply by 2

Add 9

- Then make the following assignment.

With your partner

- Choose a rule or make one up.
- Record the rule on the back of your paper.
- Make a T-chart on the front of your paper that has at least 8 examples of your pattern/rule.
- Once the groups finish allow each to share their T-chart following the procedures learned on day 2. After the students have gotten four correct responses then have the students write the rule and then the equation.

Collect all papers to use another day for the next assignment.

Part 4

Use the T-charts to create graphs.

*Notes and textbook
references*

Notes and textbook references

C. Rhombus Figures Using pattern blocks, students will build the figures below. See Blackline Master V - 7.

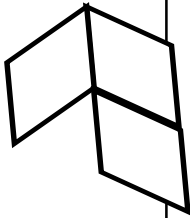


Figure #1

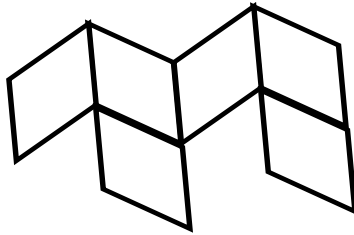


Figure #2

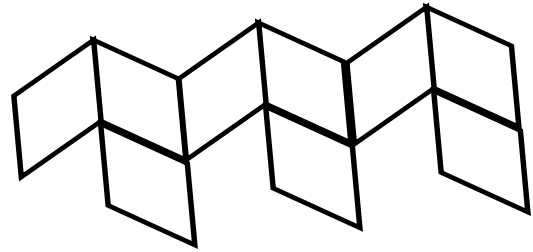


Figure #3

- How many rhombuses did you need to build each figure?
Complete the chart.

Figure Number	Number of Rhombuses
1	
2	
3	
4	
5	

- How many rhombuses would you need to make each figure?
 - Figure 8 _____
 - Figure 10 _____
 - Figure 20 _____
 - Figure 100 _____
- Write a sentence to describe the pattern between the figure number and the number of rhombuses.
- If a figure uses 36 rhombuses, what is its figure number? _____

b) Order of operations and the identity, commutative, associative, and distributive properties.

Notes and textbook references

A. Use blocks, tiles, cubes or paper squares to model expressions such as these:

$$\begin{array}{rcl} (2 \times 3) + (2 \times 2) & = & 2 \times (3 + 2) \\ 6 + 4 & = & 2 \times 5 \quad \text{(Distributive Property)} \\ 10 & = & 10 \end{array}$$

$$\begin{array}{rcl} (3 \times 4) \times 2 & = & 3 \times (4 \times 2) \\ 12 \times 2 & = & 3 \times 8 \quad \text{(Associative Property)} \\ 24 & = & 24 \end{array}$$

Use arrays and different colored markers to also represent the two expressions. Allow students to find different ways to model the expressions with their choice of manipulatives. Encourage them to also write story contexts that match the expressions.

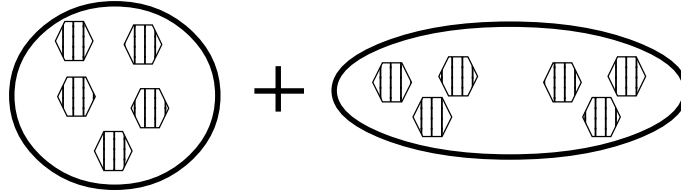
This spatial representation of the stages of the order of operations may be useful for students who are attempting to employ the order in a left to right sequence. The “floors” give equal importance to the operations appearing together.

B. Ask students to compare the models for each of the expressions. Why did they get different solutions? In order to allow students to make sense of the use of the Order of Operations, facilitate a discussion about the mathematical need for consistency in arriving at one result. Compare the need to the use of international symbols such as the choking sign, the stop sign, or the peace sign.

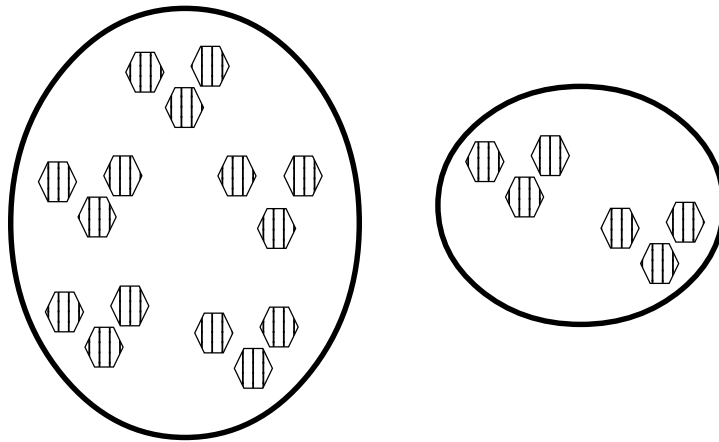
Encourage students to enter the expressions into their calculators. Discuss the results and compare to their models.

C. Try problems such as $4 \times 17 \times 25$, $2 \times 14 \times 5$ or $13 \times 8 \times 5$ to illustrate the power of the commutative and associate properties.

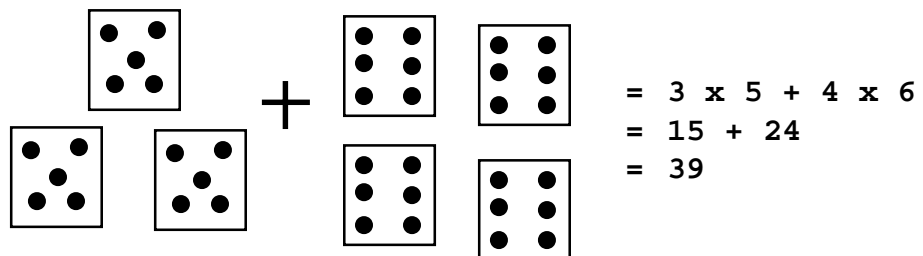
D. After sharing the memory device, “Please excuse my dear aunt Sally for looking so rough” with students, the logic of the order of operations needs to be discussed. Examples such as $5 + 2 \times 3$ and $(5 + 2) \times 3$ can be illustrated with stories or pictures. $5 + 2 \times 3$ or two groups of three plus five is eleven.



However, $(5 + 2) \times 3$ means seven groups of three or twenty-one.



E. Model the following expression.



Have the students work in pairs and create models for these and similar expressions:

- $6 + 2 \times 3 + 5$
- $6 + 2 \times (3 + 5)$
- $(6 + 2) \times 3 + 5$

F. Students take 7 by 8 block rectangular grids printed on pieces of paper. They each cut along any one of the 7 block-long segments to produce two new rectangles, for example, a 7 by 6 and a 7 by 2 rectangle. They then discuss all of the different rectangle pairs they produced and how they are all related to the original one.

G. Students often apply the mnemonic PEMDAS, in six stages rather than four. The order:

- (1) **P** parentheses implies any grouping symbol (), { }, [] ,
- (2) **E** exponents;
- (3) **MD** multiplication and division from left to right;
- (4) **AS** addition and subtraction from left to right.

The latter two stages are sometimes applied as if multiplication (or addition) is to be completed before the division (or subtraction) is attempted. The products and quotients (sums and differences) are computed in order as they appear.

$$\frac{\text{numerator}}{\text{denominator}};$$

Examples:

$27 \div 9 \times 3$ is not $27 \div 27$ or 1 but is 3×3 or 9.

$25 - 6 + 4$ is not $25 - 10$ or 15 but is $19 + 4$ or 23.

H. Play **Bowl ‘em Over!** See Blackline Master V - 3. Teacher or student spins the spinner four times. The numbers selected are the only ones students can use in a round. The object is to write equations using the four operations, at least three of the numbers from the spinner, and the correct application of the order of operations to arrive at the numbers from zero to ten. (Each correct equation knocks over one pin.) At the end of a round students record how many pins they “knocked over”. After five rounds the winner(s) are the ones with the highest score.

Example: Teacher spins 1, 3, 4, 9. A student knocking over all ten pins might write:

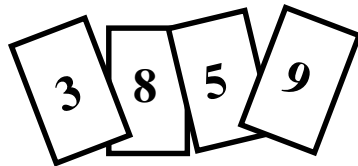
$0 = (4 - 1) - 9 \div 3$	$1 = (4 - 3) \times 1$	$2 = 9 \div 3 - 1$
$3 = 9 - (4 + 3) + 1$	$4 = 9 - (4 + 1)$	$5 = 9 \times 1 - 4$
$6 = (9 - 4) + 1$	$7 = 4 + 9 \div 3$	$8 = 9 \div 3 + 4 + 1$
$9 = 9 \times 3 \div (4 - 1)$	$10 = 9 + 1 \times (4 - 3)$	

I. Play Score! Teacher selects a target number for students to reach. Using only the numbers from one to five, the four operations and the correct application of the order of operations, students write equations which result in the target number.

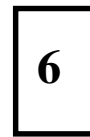
Example: Target is 14. Student writes $(5 + 3) \times 2 - 4 \div 2 = 14$. A hit!
Students can check each other's work with a calculator. More than one hit is possible in some cases. Score one point for each hit.

J. Play Match My Number! (See Blackline Master I - 1 for cards.) In pairs, students have one set of cards apiece (0 - 9). First player shuffles his/her cards and deals 4 cards (without looking). His/her opponent deals one card in the same manner. The first player must use his/her four cards and the correct order of operations to write an equation that equals the card drawn by his/her opponent. One point is awarded for a successful equation. Calculators can be used to check work. Players then return their cards to their decks and the second player repeats the process. The first player to score ten points is the Matchmaker!

Example First player draws:



Second player draws:



First player writes: $8 \div (9 - 5) \times 3 = 6$

K. Using the same cards 0 - 9, players draw four cards each from their decks. The object is to use these numbers, the operations and the correct order of operations and write an equation with the greatest (or smallest, or closest to 100, or some other target) number. A spinner has these challenges that all players strive for. After each turn the cards are shuffled and the process continues. The first player to get five points is the winner. See Blackline Master I - 1.